Practice Questions for Midterm Exam

Economic Growth
Spring 2005

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Midterm Exam from Fall 2004

Part I. (45 points) Consider the **Robinson Crusoe** model. The harvest function is given by

$$Y(t) = F\left[K(t), N(t)\right],$$

where F has the usual properties. A fraction δ of the existing trees die at each point in time. The rate of population growth is $n > 0$, and the Crusoe household has the usual utility function

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}.$$

Everything is the same as the model we studied in class, with the following **exception**. Now, at each point in time, ϕ coconuts float from the ocean onto the beach of Crusoe's island. These coconuts are not harvested, and no labor is required to collect them. Every morning, Crusoe simply finds ϕ "free" coconuts outside his house. The question we want to answer is how many of these "free" coconuts Crusoe will consume and how many he will plant. Answer this question using the following steps.

a) (5 points) Write down the optimal growth problem. Be sure to include all of the constraints.

b) (3 points) Write the Hamiltonian function for this problem and the corresponding first-order conditions. Also write the transversality condition.

c) (4 points) Using your answers above, derive a pair of differential equations for the variables (k, c).

d) (3 points) What is the economic interpretation of the multiplier μ in the Hamiltonian function? [Give a **short** intuitive answer.]

e) (10 points) Do the following comparative dynamics exercise: In the baseline economy, no coconuts arrive from the ocean ($\phi = 0$). In the modified economy, a positive number of coconuts arrive ($\phi > 0$). As usual, the baseline economy starts in its steady state, and the modified economy starts with the same number of trees per capita as the baseline economy.

Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to **label your diagram clearly**. If necessary, assume the substitution effect is larger than the income effect.

f) (10 points) Draw the time paths of c and k for both the baseline and the modified economy. Be sure to **label the curves and slopes** on your diagrams clearly.

g) (10 points) What fraction of the "free" coconuts is consumed? In other words, how does ϕ

affect the level of consumption? Why? [Give an intuitive explanation.]

Part II. (55 points) Consider the competitive equilibrium Ramsey model **with** productivity growth. The production function is given by

$$Y(t) = F\left[K(t), A(t)\, L(t)\right],$$

where F has the usual properties. Capital depreciates at rate δ, and the level of productivity $A(t)$ grows at the constant rate $g > 0$. The representative household has the usual utility function. Assume there is no population growth $(n = 0)$.

Suppose the government **taxes** only the **interest income** of households. Let τ be the tax rate, which is constant over time. The household's wage income is not taxed. Find the competitive equilibrium of this economy, using the following steps.

a) (5 points) Write down the household's complete optimization problem. Be sure to include all of the constraints.

b) (5 points) Use the Hamiltonian method to derive the household's Euler equation (the differential equation for c).

The problem of the representative firm is standard, and leads to the first-order conditions:

$$F_K(k(t), A(t)) = R(t) \quad (5)$$

$$F(k(t), A(t)) - k(t)\, F_K(k(t), A(t)) = w(t) \quad (6)$$

[You do **not** need to derive these; I am giving them to you.]

c) (5 points) What are the equilibrium conditions for this economy?

d) (5 points) Using your answers above, derive a pair of differential equations for the variables (k, c).

e) (5 points) Define the variables $\hat{k}$ and $\hat{c}$ as in class. Derive the differential equations for these variables.

f) (10 points) Do the following comparative dynamics exercise: the baseline economy has $\tau > 0$, and the modified economy has $\tau = 0$ (no tax). As usual, the baseline economy starts in the steady state at time $t = 0$. The modified economy starts at time $t = 0$ with the same amount of capital and same level of productivity as the baseline economy.

Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. If necessary, assume the **substitution** effect is larger than the income effect.

g) (10 points) Draw the time paths of c and k for both the baseline and the modified economy. Be sure to label the curves and slopes on your diagrams clearly.

h) (10 points) Do the effects of a tax on interest income more closely resemble those of a tax on **all income** or a tax on **consumption**? Why? [Give an intuitive explanation. Recall that a tax on all income and a tax on consumption are the two policies we studied in class.]

Midterm Exam from Spring 2004

Part I. (40 points) Consider the Robinson Crusoe optimal growth model. The harvest function is given by

$$Y(t) = F(K(t), N(t)),$$

where $K(t)$ is the number of trees at time t, $N(t)$ is the number of people in the household, and F has the usual properties. At every point in time a fraction δ of the existing trees die. The rate of population growth in the household is n. The utility function has the usual form (constant intertemporal elasticity of substitution)

$$u(c(t)) = \frac{c(t)^{1-\theta} - 1}{1-\theta}.$$

Everything in the model is the same as what we saw in class, with the following exception. In class we assumed that every coconut planted in the ground became a new tree. Now, only a fraction $\sigma < 1$ of the coconuts that are planted become trees. (The remaining fraction $(1-\sigma)$ of planted coconuts “die” before they grow into a tree.)

a) (10 points) Write down the complete optimal growth problem. Be sure to include all of the constraints.

b) (5 points) Write the Hamiltonian function associated with this problem. Write the first-order and transversality conditions for this problem.

c) (5 points) Use your answer from part (b) to derive a set of differential equations in the variables c and k.

d) (10 points) Do the following comparative dynamics exercise: $\sigma' < \sigma$. As usual, the baseline economy (with the parameter σ) starts in the steady state at time $t = 0$. The modified economy (with σ') starts at time $t = 0$ with the same amount of capital as the baseline economy.

Draw the phase diagrams for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. If necessary, assume the **substitution** effect is larger than the income effect.

e) (10 points) Draw the time paths of k and of c for both the baseline and the modified economy. Be sure to label the curves and slopes on your diagrams clearly.

Part II. (50 points) Consider the competitive equilibrium Ramsey model **with** productivity growth. The production function is given by

$$Y(t) = F\left[K(t), A(t)\, L(t)\right],$$

where F has the usual properties. Capital depreciates at rate δ, and the level of productivity $A(t)$ grows at the constant rate $g > 0$. The representative household has the same utility function as in Part I, and the rate of population growth is again $n > 0$.

Suppose that the government places an "asset tax" on banks: At each point in time, for each unit of machinery that the bank owns, it must pay the government an amount τ of goods.

Find the competitive equilibrium of this economy, using the following steps. First, the tax has no effect on the representative household's problem, so the equations that characterize the solution to the representative household's problem are the same as we derived in class:

$$\begin{aligned} \dot{c}(t) &= \frac{1}{\theta}\left[r(t) - \rho\right] c(t) && (1) \\ \dot{a}(t) &= w(t) + r(t)\, a(t) - c(t) - na(t) && (2) \\ \lim_{t\to\infty} \mu(t)\, a(t) &= 0 && (3) \\ a(t) &\geq -B \text{ for all } t && (4) \end{aligned}$$

(You do not have to derive these; I am giving them to you.)

a) (7 points) Write down the profit-maximization problem of a typical firm and the first-order conditions for this problem. Convert these equations to intensive form.

b) (3 points) What are the equilibrium conditions for this economy?

c) (3 points) Using your answers above, derive a pair of differential equations for the variables (k, c).

d) (7 points) Define the variables $\hat{k}$ and $\hat{c}$ as in class. Derive the differential equations for these variables.

e) (10 points) Do the following comparative dynamics exercise: the baseline economy has $\tau > 0$, and the modified economy has $\tau = 0$ (no tax). As usual, the baseline economy starts in the steady state at time $t = 0$. The modified economy starts at time $t = 0$ with the same amount of capital and same level of productivity as the baseline economy.

Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. If necessary, assume the **income** effect is larger than the substitution effect.

f) (10 points) Draw the time paths of c and k for both the baseline and the modified economy. Be sure to label the curves and slopes on your diagrams clearly.

g) (10 points) In the long run (as $t \to \infty$), what is the interest rate that households receive on their savings in (i) the baseline economy and (ii) the modified economy? Based on your answer, is this model consistent with the evidence on Capital Flows discussed in class?

Part III. (10 points) Consider again the Ramsey model with the production function

$$Y(t) = F\left[K(t), A(t)\, L(t)\right].$$

As above, the level of productivity $A(t)$ grows at the constant rate $g > 0$ and the population grows at the constant rate $n > 0$. We know that in the long run, the economy converges to a balanced growth path.

a) (1 point) Along a balanced growth path, what is the growth rate of the variables k and y?

b) (3 points) Along a balanced growth path, what are the growth rates of the variables Y, K and L? Notice that these variables are not in intensive form. (The answer may be different for each of these variables.)

c) (6 points) Suppose that a group of economists does the standard growth accounting exercise for this economy. They use the production function

$$Y(t) = B(t)\, K(t)^{\alpha}\, L(t)^{1-\alpha}$$

for their analysis, where $B(t)$ is total factor productivity. They observe only the growth rates for Y, K and L given in your answer to (b). What is the growth rate of total factor productivity that they will compute?