Problem Set #2

Economic Growth
Spring 2005

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1) Consider the optimal growth problem with the CIES utility function

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}$$

and the Cobb-Douglas production function $Y(t) = BK(t)^{\alpha} N(t)^{1-\alpha}$, where $B > 0$ is a constant. Assume $n > 0$.

a) Write down the complete optimal growth problem. Solve this problem using the Hamiltonian method and derive two differential equations in the variables (c, k). This is very similar to the second question on Problem Set #1; the only difference is the constant B in the production function.

b) Do the following comparative dynamics exercise: $B' > B$, following the usual steps. Draw the phase diagram for the baseline case, and suppose that k_0 is equal to k^* for this case. Then draw the modified phase diagram, indicating what has changed with the higher value of B. Draw the modified time paths of k and c, indicating how they compare with the baseline time paths. Pay special attention to the value of c at $t = 0$; is it higher or lower in the modified case? Why? (Give an intuitive answer.)

2) Consider the Ramsey model of an economy in competitive equilibrium. There is a representative household and a representative firm. Assume there is no population growth $(n = 0)$. The household's utility function is

$$\int_0^{\infty} \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt,$$

and the firm has a constant-returns-to-scale production function $Y(t) = F[K(t), L(t)]$. Everything is the same as what we saw in class, with the following exception. In class, we assumed that one unit of output that is not consumed becomes one unit of capital. Now, one unit of output that is not consumed becomes $\sigma < 1$ units of capital. We can think of the parameter σ as measuring the efficiency of the financial sector. One of the primary roles of the financial sector is to put the savings of households to productive use. When σ is low, it takes a lot of saving to create one new productive machine. In this sense, we can say that the financial sector is inefficient.

Find the competitive equilibrium of this economy, using the following steps. [Hint: These steps are the same ones we have gone through before. At each step, ask: Should σ appear here? Remember that the assets of the household $\mathcal{A}(t)$ is measured in units of goods, not in units of capital. Pay particular attention to **both** of the equations related to banks. The profits of banks should be measured in units of goods, not in units of capital.]

a) Write down representative household's maximization problem and derive the 4 equations that characterize the solution.

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

c) What are the equilibrium conditions for this economy?

d) Combine your answers to parts (a) - (c) and derive a pair of differential equations for the variables c and k.

e) Do the following comparative dynamics exercise: $\sigma' > \sigma$. As usual, the baseline economy starts in its steady state at time $t = 0$. The modified economy starts at time $t = 0$ with the same amount of capital as the baseline economy. Draw (i) the phase diagram for both cases, indicating what is different, and (ii) the time paths of c and k for both cases. If necessary, assume that the substitution effect dominates the income effect.