

## Problem Set #4

Economic Growth  
Spring 2005

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### No Due Date

- 1) Consider the Ramsey model **with** productivity growth. Suppose the government imposes a sales tax on firms: the representative firm must pay a fraction  $\tau$  of its output to the government. Assume there is population growth ( $n > 0$ ). Find the competitive equilibrium of this economy, using the following steps.
- Write down the 4 equations that characterize the optimal behavior of the representative household.
  - Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.
  - What are the equilibrium conditions for this economy?
  - Let  $\phi(t)$  denote the per-capita revenue of the government at time  $t$ . What is the government's budget constraint?
  - Combine your answers to parts (a) - (d) to get a pair of differential equations for the variables  $k$  and  $c$ .
  - Translate these two equations into per-effective-workers terms (so that we have differential equations for the variables  $\hat{k}$  and  $\hat{c}$ ).
  - Do the following comparative dynamics exercise:  $\tau' < \tau$ . As usual, assume that  $k_0$  and  $A_0$  are the same for both economies, and are such that the baseline economy starts in its steady state at time  $t = 0$ . Draw (i) the phase diagram for both cases, indicating what is different, (ii) the time paths of  $\hat{k}$  and  $\hat{c}$  for both cases, and (iii) the time paths of  $k$  and  $c$  for both cases. If necessary, assume that the substitution effect dominates the income effect.
- 2) Consider the following data for Mexico in 1965 and 1990:

Year	Population	Real GDP	Labor Force	Capital Stock
1965	44,854,000	150,305,754,000	13,029,278	25,186,454,808
1990	81,724,000	476,205,748,000	27,992,344	81,969,979,536

- What is the average annual growth rate of real GDP over this period? Do the growth accounting analysis using this data, and using  $\alpha = 0.69$ . [The data is drawn from the Penn World Table, ver. 5.6. The Real GDP and Capital Stock variables are measured in 1985 US\$. The Labor Force variable is an estimate of the number of full-time equivalent workers employed during the year. The Capital Stock variable is an estimate of the value of all producer durables.]

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b) In class, we saw the following information for the period 1940-1980 (taken from p. 381 of Barro & Sala-i-Martin):

1940-1980 :	GDP	Capital	Labor	TFP
Growth rate	0.0630	0.0370	0.0468	
Contribution		0.0255	0.0145	0.0230
Fraction of total contribution		(40.5%)	(23.0%)	(36.5%)

How do the results for the two time periods compare? In particular, to what factor does the analysis attribute most of the slowdown of GDP growth in Mexico?