# *Problem Set #5*

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In this problem, you will examine the effect of aid for development when the differences between economies are caused by differences in the efficiency of the financial sector. The production function is the usual

$$Y(t) = F\left[K(t), A(t)L(t)\right],$$

where the level of productivity $A(t)$ grows at the constant rate $g > 0$. Everything is standard with two exceptions:

$(i)$ One unit of output that is not consumed becomes $\sigma < 1$ units of capital. As we have discussed before, we can think of $\sigma$ as measuring the efficiency of the financial sector (that is, how efficiently saving is transformed into productive investment).

$(ii)$ At every point in time $t$, the economy receives $\phi A(t)$ units of **capital** as an aid payment.

a) Write down the complete optimal growth problem. That is, we are going to solve the Pareto problem. We are *not* going to solve for the competitive equilibrium. [Of course, the two approaches will lead to the same answer here.]

b) Write the Hamiltonian function associated with this problem. Use the first-order conditions for this problem to derive a pair of differential equations in the variables $c$ and $k$.

c) Define the variables $\hat{k}$ and $\hat{c}$ as usual. Derive the differential equations for these variables.

d) Do the following comparative dynamics exercise: The baseline economy has $\phi = 0$ (no aid) and the modified economy has $\phi > 0$. As usual, the baseline economy starts in the steady state for $\left(\hat{k}, \hat{c}\right)$ at time $t = 0$. The modified economy starts at time $t = 0$ with the same amount of capital and the same level of productivity as the baseline economy.

Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. If necessary, assume that the substitution effect is larger than the income effect.

e) Draw the time paths of $k$ and $c$ for both the baseline and the modified economy. Be sure to label the curves and slopes on your diagrams clearly. The diagrams show the predicted effect of aid for development.

f) Now do a different comparative dynamics exercise (still using your answer to part (c)): $\sigma' > \sigma$. Assume $\phi = 0$ in both cases. In other words, the economy is now receiving no aid, and we are examining the predicted effect of an increase in the efficiency of the financial sector (that might result from some sort of reform). As usual, the baseline economy starts in the steady state for

$\left(\hat{k}, \hat{c}\right)$ at time $t = 0$. The modified economy starts at time $t = 0$ with the same amount of capital and the same level of productivity as the baseline economy.

Draw the phase diagram for both the baseline and the modified economy, indicating what is different. Be sure to label your diagram clearly. If necessary, assume that the substitution effect is larger than the income effect.

g) Draw the time paths of $k$ and $c$ for both the baseline and the modified economy. Be sure to label the curves and slopes on your diagrams clearly.

[The point of this exercise is to compare your answers to (e) and (g). According to this model, which is more effective in terms of encouraging economic growth: aid or financial reform?]