Problem Set #7

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Consider the Ak model, where there are externalities in production. As in class, the production function of the representative firm is given by

$$Y = AK^\alpha L^{1-\alpha}\overline{K}^{1-\alpha},$$

where $\overline{K}$ is the total amount of capital in the economy. Consumers have the usual utility function

$$u(c) = \frac{c^{(1-\theta)} - 1}{1 - \theta}.$$

Assume there is no population growth $(n = 0)$, and normalize the population to $N = 1$.

We saw in class that the government could make the equilibrium optimal by providing a subsidy on capital rental by firms. Suppose that the government tries a different policy: it subsidizes *savings by households*. In particular, for each unit of assets that the household owns at time t, the government gives the household a payment of σ (this is in addition to the payment $r(t)$ that the household receives from the bank). Assume that σ is constant through time. To finance this expenditure, the government taxes labor income at time t at rate $\tau(t)$.

a) Write the household's maximization problem and derive the differential equations for the variables c and a.

b) Write the maximization problem of a typical firm and solve this problem to obtain the rental rate and wage as functions of k and $\bar{k}$.

c) What are the equilibrium conditions for this economy?

d) Assume the government has a balanced budget at each point in time. What is the government's budget constraint?

e) Use the information from the previous parts to derive equilibrium differential equations for the variables c and k. (Note: these equations may depend on the level of subsidy σ).

f) Can the level of subsidy σ be chosen so that the equilibrium is optimal? (You do not need to solve the optimal growth problem; just use the results from class.) If so, what value of σ does this? What is the implied tax rate on labor income $\tau(t)$?