Banking and Financial Fragility

Financial Contagion: Allen and Gale (2000)

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May 2017

Contagion

- Financial crises often spread very quickly
  - problems may start in one region or one institution
  - but often trigger runs on other (unrelated?) institutions or in other regions
- Why?
- The Diamond-Dybvig model provides one theory
  - suppose Bank A fails (for whatever reason)
  - if this event causes investors elsewhere to lose confidence in their own banks …
  - … they may decide to withdraw …
  - and the belief that the crisis will spread becomes self-fulfilling

- According to this view, a crisis may spread …
- But it also may not spread
  - suppose investors in other banks do not lose confidence
- Allen & Gale show us how the situation may be worse than this view indicates
  - framework is very close to Diamond & Dybvig, but with multiple banks
  - under some conditions, a run on one bank <u>must</u> lead to runs on the other banks ⇒ “true” contagion
- Readings:
  - Allen & Gale (JPE, 2000)
  - Allen & Gale book, chapter 10

Outline

1. The Environment with Two Regions
2. The Efficient Allocation
3. Banking
4. Fragility and Contagion
5. Many Regions
6. Summary

1. The Environment with Two Regions

- The same as in our Diamond-Dybvig model, except:
- There are now two locations: A, B
  - each with a [0,1] continuum of investors
- There is uncertainty about the fraction of investors in each location who are impatient

	Location		
state	A	B	probability
s_1	λ_H	λ_L	1/3
s_2	λ_L	λ_H	1/3
s_3	$\bar{\lambda}$	$\bar{\lambda}$	1/3

  - where $\lambda_H > \lambda_L$ and $\bar{\lambda} = \frac{\lambda_H + \lambda_L}{2}$

2. The (full information) efficient allocation

2.1 The planner's problem

- Suppose a planner could observe investors' types and control resources in both locations
- Note: there is no *aggregate* uncertainty about λ
  - uncertainty is about where impatient investors will be located
- Some properties of any efficient allocation:
  - no investment should be liquidated at $t = 1$
  - no storage should be held until $t = 2$

  } as before
- In state s_1, for example:

$$\lambda_H c_1^A(s_1) + \lambda_L c_1^B(s_1) = 2(1 - x)$$
$$(1 - \lambda_H) c_2^A(s_1) + (1 - \lambda_L) c_2^B(s_1) = 2Rx$$

- Repeating:

$$\lambda_H c_1^A(s_1) + \lambda_L c_1^B(s_1) = 2(1-x)$$
$$(1-\lambda_H)c_2^A(s_1) + (1-\lambda_L)c_2^B(s_1) = 2Rx$$

- Suppose the planner wants to set $c_t^A(\mathrm{s}) = \mathrm{c}_t^B(\mathrm{s})$ for all t, s
  - that is, planner treats investors in both banks equally

$$\left.\begin{aligned} \frac{\lambda_H+\lambda_L}{2} c_1(s_1) &= (1-x) \\ \left(1-\frac{\lambda_H+\lambda_L}{2}\right) c_2(s_1) &= Rx \end{aligned}\right\}$$

$\Rightarrow c_1$ and c_2 are independent of s

- So we have

$$\left.\begin{aligned} \bar{\lambda} c_1 &= 1-x \\ (1-\bar{\lambda}) c_2 &= Rx \end{aligned}\right\} \Rightarrow \bar{\lambda} c_1 + (1-\bar{\lambda})\frac{c_2}{R} = 1$$

as in the baseline model (!)

- Investors' expected utility from (c_1, c_2):

$$\frac{1}{3}(\lambda_H u(c_1) + (1-\lambda_H)u(c_2)) + \frac{1}{3}(\lambda_L u(c_1) + (1-\lambda_L)u(c_2)) + \frac{1}{3}\left(\bar{\lambda}u(c_1) + \left(1-\bar{\lambda}\right)u(c_2)\right)$$

- Note: $$\frac{1}{3}\lambda_H + \frac{1}{3}\lambda_L + \frac{1}{3}\bar{\lambda} = \bar{\lambda}$$

- The planner would then choose (c_1, c_2) to solve

$$\max_{\{c_1,c_2\}} \bar{\lambda}u(c_1) + \left(1-\bar{\lambda}\right)u(c_2)$$

subject to $$\bar{\lambda}c_1 + \left(1-\bar{\lambda}\right)\frac{c_2}{R} = 1$$

solution: (c_1^*, c_2^*)

Two key points:

(a) It is *feasible* for the planner to give the consumption plan (c_1^*, c_2^*) to every investor in every state

- because there is no aggregate uncertainty

(b) If the planner places equal weight on all investors, then (c_1^*, c_2^*) is the *optimal* allocation

more intuition more details

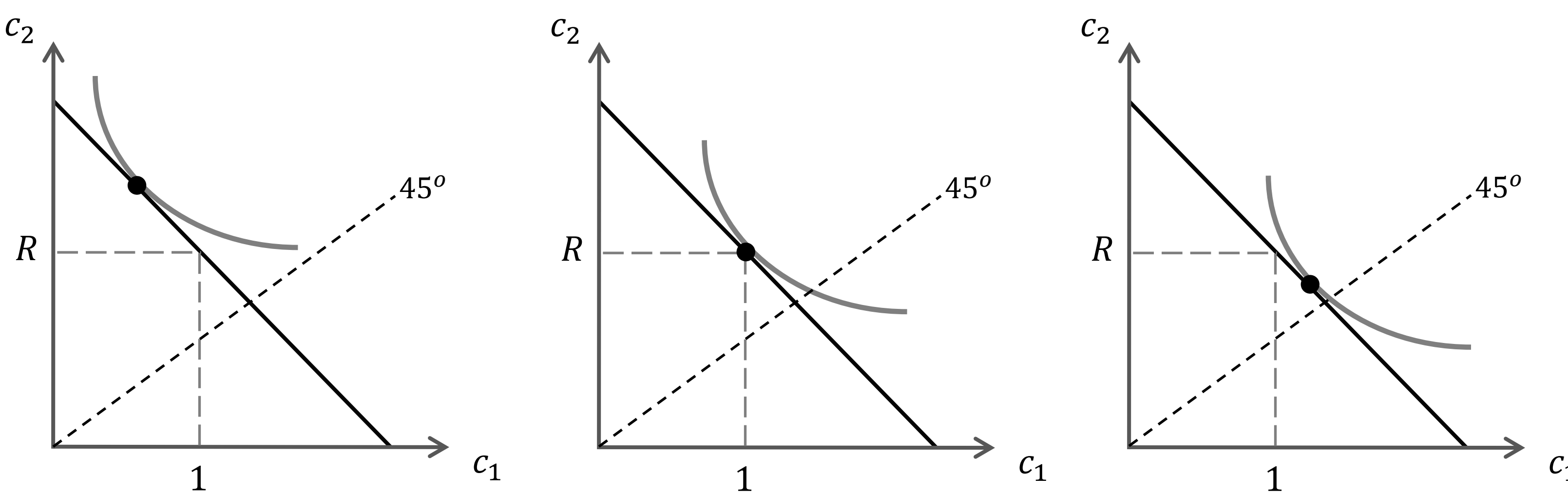
In other words:

- The planner sees one big Diamond-Dybvig economy
  - the regions are not relevant from the planner's point of view
  - desired allocation of resources is exactly the same as before

- The efficient allocation is again summarized by two numbers:

$$(c_1^*, c_2^*) \text{ with } \quad c_1^* < c_2^*$$

- Possibilities:



2.2 Regional transfers

- A key feature of this allocation:
  - the planner must transfer resources across regions
- Suppose the same portfolio is used in both regions

$$1 - x = \bar{\lambda} c_1^*$$

$$x = (1 - \bar{\lambda}) \frac{c_2^*}{R}$$

- When a region has λ_H impatient investors, it needs more resources at $t = 1$
  - these resources come from storage in the other region, where there are only λ_L impatient investors
  - the λ_H region then has *extra* resources at $t = 2$

- At $t = 1$:

	state s_1		state s_2	
	$\underline{A}$	$\underline{B}$	$\underline{A}$	$\underline{B}$
storage:	$\bar{\lambda} c_1^*$	$\bar{\lambda} c_1^*$	$\bar{\lambda} c_1^*$	$\bar{\lambda} c_1^*$
impatient consumption:	$\lambda_H c_1^*$	$\lambda_L c_1^*$	$\lambda_L c_1^*$	$\lambda_H c_1^*$
	$\longleftarrow$		$\longrightarrow$	
transfer of:	$(\lambda_H - \bar{\lambda}) c_1^*$		$(\lambda_H - \bar{\lambda}) c_1^*$	

- At $t = 2$:

	state s_1		state s_2	
	$\underline{A}$	$\underline{B}$	$\underline{A}$	$\underline{B}$
matured investment:	$(1 - \bar{\lambda}) c_2^*$	$(1 - \bar{\lambda}) c_2^*$	$(1 - \bar{\lambda}) c_2^*$	$(1 - \bar{\lambda}) c_2^*$
patient consumption:	$(1 - \lambda_H) c_2^*$	$(1 - \lambda_L) c_2^*$	$(1 - \lambda_L) c_2^*$	$(1 - \lambda_H) c_2^*$
	$\longrightarrow$		$\longleftarrow$	
transfer of:	$(\lambda_H - \bar{\lambda}) c_2^*$		$(\lambda_H - \bar{\lambda}) c_2^*$	

- These inter-region transfers are the new element in the Allen-Gale model
- At the aggregate level: everything is the same as before
  - the overall economy is exactly as in Diamond & Dybvig
- But there is now uncertainty at the regional level
  - result: the efficient allocation requires transferring resources across regions in each period
- How can our banking arrangement generate these transfers?
  - need to somehow include them in the rules governing bank behavior

3. Banking

3.1 A banking arrangement

- Assume one (representative) bank per region
- Each offers investors the same contract as before …
  - collects deposits at $t = 0$
  - allows investors to choose when they withdraw
  - withdrawals at $t = 1$ are paid c_1^* as long as funds are available
- … and invests according to average liquidity demand:

$$1 - x = \bar{\lambda} c_1^*$$

$$x = \left(1 - \bar{\lambda}\right) \frac{c_2^*}{R}$$

Interbank deposits:

- At $t = 0$, Bank A deposits an amount z in Bank B
- … and Bank B deposits z in Bank A
- Interbank deposits have same rules as investor deposits
  - can be withdrawn in either period
  - withdrawing bank receives zc_1^* at $t = 1$ if funds are available
  - or a z-share of other bank's assets at $t = 2$
- Note: total funds available at $t = 0$ in Bank A:

$$1 + z - z = 1$$

- Assume each bank deposits with the other bank:

$$z = (\lambda_H - \bar{\lambda}) \qquad \left(= \left(\bar{\lambda} - \lambda_L\right) = \frac{\lambda_H - \lambda_L}{2}\right)$$

- To meet withdrawals at $t = 1$, a bank will:
  - first use resources in storage,
  - then withdraw its interbank deposit,
  - then liquidate investment

"liquidation pecking order"

- A bank withdraws its interbank deposit if and only if $t = 1$ withdrawals exceed $\bar{\lambda} c_1^*$

- As before, the banking rules create a withdrawal game
- Players: the investors in both regions
  - banks are non-strategic; they simply follow the specified rules
- Timing:
  - investors observe state s at the very beginning of $t = 1$
  - before choosing a withdrawal strategy
- We will study the game *separately in each state*
  - simplifies the notation, with no loss of generality
  - investors observe state s, then play the withdrawal game associated with s

3.2 Strategies

- As before: impatient investors always withdraw at $t = 1$
  - do not value consumption at $t = 2$
- A strategy for an investor in Bank j is

$$y_i^j \in \{1,2\} \qquad \text{as before}$$

  - $y_i = t$ means withdraw in period t *when patient*
- Other notation is similar to before:
  - $y = \left\{y_i^j\right\}_{j\in\{A,B\},i\in[0,1]}$ is a profile of withdrawal strategies
  - y_{-i} = strategies of all investors (in both banks) except i

- For any y_{-i}, define:

$$e_j(y_{-i}) = \text{number of } t = 1 \text{ withdrawals by patient investors in bank } j \in \{A, B\}$$

  - as before: $e_j \in [0, 1 - \lambda]$

- Rather than fully deriving the best-response functions, we will look for particular types of equilibria
  - ask whether certain profiles y are an equilibrium of the game

3.3 Equilibrium

Q: Is there an equilibrium with

$$y_i^j = 2 \quad \forall\, i, \forall\, j\,?$$

- Suppose y_{-i} has this form.
  - then $e_A(y_{-i}) = e_B(y_{-i}) = 0$
- Focus on the payoffs of investor i in state s_1
  - withdraws at $t = 1 \Rightarrow$ receives c_1^*
  - withdraws at $t = 2 \Rightarrow$ receives even share of her bank's assets
- What is this even share worth?

- In state s_1, Bank A (with λ_H) has:

$$1 - x^* \quad + \quad zc_1^* \quad - \quad \lambda_H c_1^* \quad + \quad Rx^* \quad - \quad zc_2^*$$

storage; from Bank B; to impatient investors; matured investment; to Bank B

$$= \underbrace{\bar{\lambda} c_1^* + (\lambda_H - \bar{\lambda}) c_1^* - \lambda_H c_1^*}_{=0} + \underbrace{R(1 - \bar{\lambda}) \frac{c_2^*}{R} - (\lambda_H - \bar{\lambda}) c_2^*}_{= (1 - \lambda_H) c_2^*}$$

- An even share is worth:

$$c_{2,A}(e_A = e_B = 0; s_1) = \frac{(1 - \lambda_H) c_2^*}{1 - \lambda_H} = c_2^*$$

- In state s_1, Bank B (with λ_L) has:

$$1 - x^* \quad - \quad zc_1^* \quad - \quad \lambda_L c_1^* \quad + \quad Rx^* \quad + \quad zc_2^*$$

storage | to Bank A | to impatient investors | matured investment | from Bank A

$$= \underbrace{\bar{\lambda}c_1^* - \left(\lambda_H - \bar{\lambda}\right)c_1^* - \lambda_L c_1^*}_{=0} \quad + \quad \underbrace{R\left(1 - \bar{\lambda}\right)\frac{c_2^*}{R} + \left(\lambda_H - \bar{\lambda}\right)c_2^*}_{= (1-\lambda_L)c_2^*}$$

verify using
$(\lambda_H - \bar{\lambda}) = (\bar{\lambda} - \lambda_L)$

- An even share is worth:

$$c_{2,B}(e_A = e_B = 0; s_1) = \frac{(1 - \lambda_L)c_2^*}{1 - \lambda_L} = c_2^*$$

- Best response of an investor in either bank is then

$$y_i^j = 2$$

Result 1: There is a Nash equilibrium in state s_1 with

$$y_i^j = 2 \quad \text{for all } i.$$

- Verify: the same result holds in states s_2, s_3

- Each investor receives consumption plan (c_1^*, c_2^*)
  - in every state of nature
  - even though state is not known when investment decisions are made

- Result 1 demonstrates the benefits of interbank deposits
  - allow efficient transfers of storage and investment across regions
  - a form of “risk sharing”
- Similar in spirit to the first result in Diamond & Dybvig
  - showed the benefits of maturity transformation
- Next question: what can go wrong?

4. Fragility and Contagion

- Under assumption (A1), there is an equilibrium where investors run on both banks, that is

$$y_i^A = 1 \quad \text{and} \quad y_i^B = 1 \ \text{ for all } i$$

- In this equilibrium, both banks withdraw their interbank deposit at $t = 1$
  - these deposits then simply cancel out
  - the analysis is exactly the same as in Diamond & Dybvig
- In this scenario, the run on one bank is not *causing* the other bank to fail
  - why did investors in Bank B lose confidence?
  - perhaps because of the run on Bank A ("simple" contagion)
  - or perhaps for some other reason

- Want to see how a problem in one bank *affects* the other
  - suppose the problem starts in Bank A

Q: Is there an equilibrium of this game in which:

  - investors in Bank A run, but investors in Bank B do not run?

- If Bank B remains solvent, answer is "yes"
  - we will say there is "no contagion" in this case

- If the run on Bank A makes B insolvent, answer is "no":
  - the only equilibrium with a run on A also has a run on B
  - in this sense, a run on Bank A causes a run on Bank B
  - this is "contagion" in the Allen & Gale sense

- Note: with no interbank deposits, answer would be "yes"
  - if there is no relationship between the banks …
  - then the outcome at A has no direct implication for B
- With interbank deposits …
  - when Bank A fails, Bank B will lose money on its deposit
  - what are the implications for Bank B? (we need to check)
- To simplify the analysis, assume:
  - $u(c) = \ln(c) \;\Rightarrow\; (c_1^*, c_2^*) = (1, R)$
  - focus on the withdrawal game in state s_3
  - only serves to make the calculations easier

4.1 Calculating payoffs

- Suppose $y_i^A = 1$ and $y_i^B = 2$
- Then $e_A(y_{-i}) = 1 - \bar{\lambda}$ and $e_B(y_{-i}) = 0$
- What is the best response of an investor in each region?
  - does the interbank deposit make joining the run on Bank A less attractive?
  - what are the implications of the run on Bank *A* for investors in Bank *B*?
- Proceed in three steps, studying:
  i. interbank withdrawal behavior
  ii. fraction of investors served in Bank *A*
  iii. payoffs of investors in Bank *B*

Step (i): Interbank withdrawal behavior

- Recall that a bank will withdraw its interbank deposit if and only if $t = 1$ withdrawals exceed $\bar{\lambda} c_1^*$

- All investors at Bank A attempt to withdraw at $t = 1$
  $\Rightarrow$ A withdraws its deposit from Bank B
  - suppose it receives $z c_1^*$ (face value)

- Then $t = 1$ withdrawals at Bank B are:

$$(\bar{\lambda} + z) c_1^* > \bar{\lambda} c_1^*$$

($\bar{\lambda}$: impatient investors; z: Bank A)

$\Rightarrow$ Bank B withdraws its deposit from Bank A (**!**)

Step (ii): Fraction of investors served in Bank A:

$$q_A = \frac{total\ assets}{total\ withdrawals} = \frac{rx^* + (1 - x^*) + zc_1^*}{c_1^* + zc_1^*}$$

Numerator: liquidated investment (rx^*), storage ($(1-x^*)$), from Bank B (zc_1^*)

Denominator: own investors (c_1^*), to Bank B (zc_1^*)

- Using $(c_1^*, c_2^*) = (1, R)$, we have $x^* = (1 - \bar{\lambda})$ and

$$q_A = \frac{r(1-\bar{\lambda})+\bar{\lambda}+(\lambda_H-\bar{\lambda})}{1+(\lambda_H-\bar{\lambda})} = \frac{\lambda_H+r(1-\bar{\lambda})}{\lambda_H+(1-\bar{\lambda})} < 1$$

  - Bank A is bankrupt, despite the interbank deposit

- Repeating:

$$q_A = \frac{\lambda_H + r(1-\bar{\lambda})}{\lambda_H + (1-\bar{\lambda})} < 1$$

- An example:

$$r = \frac{1}{2}, \quad \lambda_H = \frac{3}{4}, \quad \lambda_L = \frac{1}{4} \quad \Rightarrow \quad \bar{\lambda} = \frac{1}{2}$$

  - then (verify)

$$q_A = \frac{4}{5} \quad \text{(80\% payout rate)}$$

- Note:

$$c_{2,A}\left(e_A = 1 - \bar{\lambda}, e_B = 0\right) = 0$$

  - best response of a patient investor in Bank A is indeed to withdraw at $t = 1$

Step (iii): Payoffs of investors in Bank B

- Assume it receives a fraction q_A of its deposit from Bank A
  - rather than receiving whole deposit with probability q_A
  - idea: deposit represents many distinct interbank exposures
- Needs $\bar{\lambda} c_1^*$ for its impatient investors, so …
  - must liquidate $\frac{(1-q_A) z c_1^*}{r}$ units of investment
  - why? To cover the losses on its interbank deposits

- An investor in Bank B who withdraws at $t = 2$ receives:

$$c_{2,B}\left(e_A = 1-\bar{\lambda}, e_B = 0; s_3\right) = \max\left\{\frac{R\left(x^* - \frac{(1-q_A)zc_1^*}{r}\right)}{1-\bar{\lambda}}, 0\right\}$$

- Using $(c_1^*, c_2^*) = (1, R)$,

$$c_{2,B}\left(e_A = 1-\bar{\lambda}, e_B = 0; s_3\right) = \max\left\{R\left(1 - \frac{(1-q_A)\left(\lambda_H - \bar{\lambda}\right)}{r\left(1-\bar{\lambda}\right)}\right), 0\right\}$$

- For our example:

$$= R\left(1 - \frac{(1-q_A)\frac{1}{4}}{\frac{1}{4}}\right) = q_A R$$

4.2 Conditions for contagion

Result 2: If $c_{2,B}(e_A = 1 - \bar{\lambda}, e_B = 0; s_3) \geq c_1^*$

then y is a Nash equilibrium in state s_3.

- in our example, this requires

$$q_A R \geq 1 \quad \text{or} \quad R \geq \frac{1}{q_A} = \frac{5}{4} \quad (= 1.25)$$ "no contagion"

- Bank B suffers losses on its deposit, but not a run

Result 3: Otherwise, y is not a Nash equilibrium in s_3.

- in this case, the only equilibrium with $y_i^A = 1$ also has $y_i^B = 1$
- a run on Bank A necessarily causes a run on Bank B

$\Rightarrow$ "financial contagion" (Allen & Gale)

- Looking at the balance sheet of Bank B
  - after liquidating investment to cover loss on interbank deposit

Assets		Liabilities	
Investment	$R\left(x^* - \frac{(1-q_A)zc_1^*}{r}\right)$	Deposits	c_1^*
Storage	$1 - x^*$		
		Equity	E

- Bank B is solvent if $E \geq 0$, or:

$$R\left(x^* - \frac{(1-q_A)zc_1^*}{r}\right) + 1 - \mathrm{x}^* \geq c_1^*$$

- Solve for: $R \geq \frac{1}{q_A}$

⇒ contagion occurs when losses make Bank B insolvent

4.3 Equilibrium payoffs

- The payoffs calculated above assumed no run on Bank B
- If the run spreads to Bank B, it fails at $t = 1$ and …
  - Bank A suffers losses on its interbank deposit
  - q_A is even lower than what we calculated above
- The fractions of investors served in equilibrium are

$$q_A = \frac{rx^* + (1 - x^*) + q_B z c_1^*}{(1+z)c_1^*}$$

$$q_B = \frac{rx^* + (1 - x^*) + q_A z c_1^*}{(1+z)c_1^*}$$

two equations in two unknowns

- Solve for

$$q_A = q_B = \frac{1 - (1 - r)x^*}{c_1^*}$$

the same as in our baseline model

- For our example:

$$q_A = q_B = \frac{3}{4} \qquad \left(< \frac{4}{5}\right)$$

- Due to the interbank deposits, the liquidation costs of a run are always shared by investors in both banks
- If only Bank A experiences a run, its investors suffer a loss of 20%
  - investors in Bank B also lose some, but less
- If the run spreads to Bank B, the losses of *Bank A's investors* increase to 25%
  - in addition, investors in Bank B now lose 25% as well

4.4 Extending the analysis to other states

	Location		
state	A	B	probability
s_1	λ_H	λ_L	1/3
s_2	λ_L	λ_H	1/3
s_3	$\bar{\lambda}$	$\bar{\lambda}$	1/3

- We have focused on state s_3 to simplify the calculations
- Now consider the withdrawal game in state s_2
- If there is a run on Bank A:
  - both banks will withdraw their interbank deposits
  - Bank A will fail, imposing losses on Bank B
  - Bank B is in worse condition than before because it has λ_H

$\Rightarrow$ the run on Bank A is *more likely* to spread to Bank B

	Location		
state	A	B	probability
s_1	λ_H	λ_L	1/3
s_2	λ_L	λ_H	1/3
s_3	$\bar{\lambda}$	$\bar{\lambda}$	1/3

- Now consider state s_1
  - note: a run on Bank B would easily spread to Bank A in s_1
- If there is a run on Bank A:
  - when does Bank B withdraw its interbank deposit?
  - Bank B does not need the funds at $t = 1$
  - but it knows that if it waits until $t = 2$ it will get nothing

$\Rightarrow$ need to extend our rules of banking to fully study this case

Bottom line (so far)

- Interbank linkages are socially useful …
  - allow diversification of bank-specific liquidity risk
- …but make financial crises contagious
  - a trigger that causes a run on one bank …
  - … could lead to the failure of many or all banks

  ⇒ small shocks can have very large consequences
- Focusing on state s_3 makes these points in the clean way
  - but the same message emerges in all three states

5. Many Regions

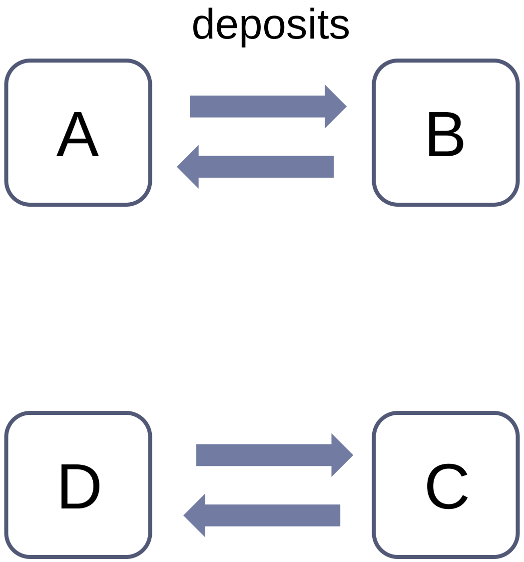
- Now suppose there are four regions, with

	Location				
state	A	B	C	D	probability
s_1	λ_H	λ_L	λ_H	λ_L	$1/3$
s_2	λ_L	λ_H	λ_L	λ_H	$1/3$
s_3	$\bar{\lambda}$	$\bar{\lambda}$	$\bar{\lambda}$	$\bar{\lambda}$	$1/3$

  - regions C and D are replicas of A and B
- Risk-sharing role of interbank deposits is the same
- But now there are different ways in which these deposits can be arranged
  - Bank A could deposit with B, with D, or with both of them

5.1 Bilateral interbank deposits

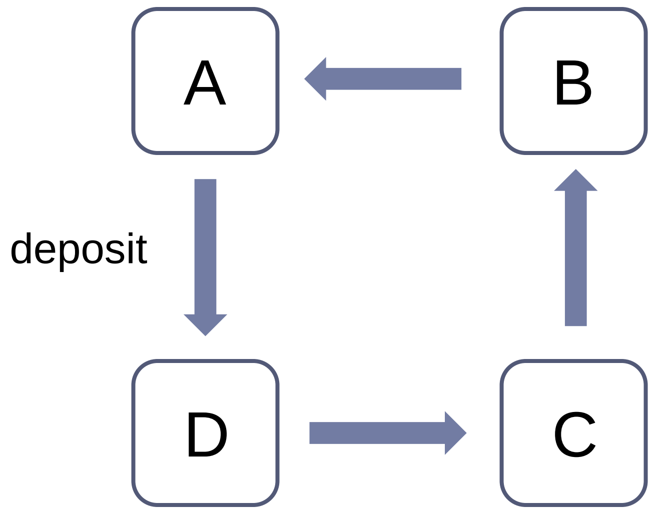
- Suppose:



- analysis is unchanged

5.1 A circular network of deposits

- Now suppose



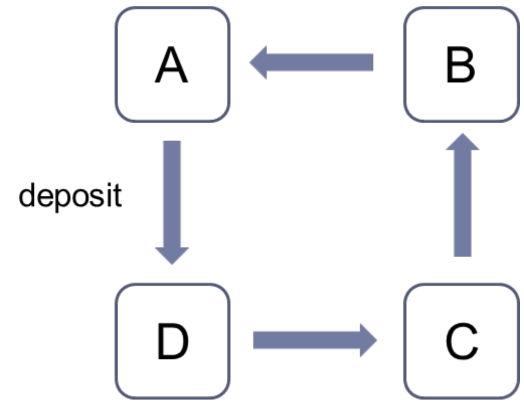
- Under this pattern there is again an equilibrium with

$$y_i^j = 2 \quad \forall\, i, \forall\, j$$

  - implements the (same) efficient allocation

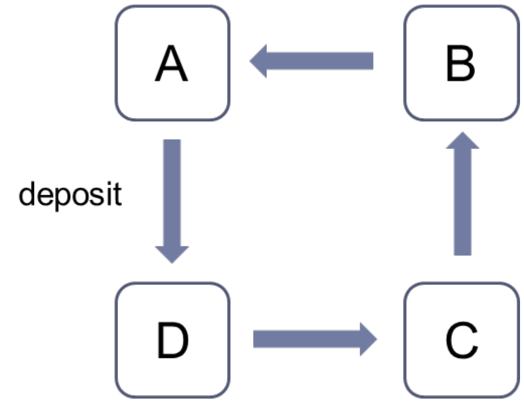
- But what happens now if there is a run on Bank A?

- Focus again on state s_3
- Suppose $y_i^A = 1$ and
  $y_i^j = 2$ for $j = B, C, D$
- Follow the same three steps as before:
  i. interbank withdrawal behavior
  ii. fraction of investors served in Bank A
  iii. payoffs of investors in Bank B



- If the run on Bank A causes Bank B to fail …
  - suppose $y_i^B = 1$, then repeat step (ii) for Bank B
  - and step (iii) for the Bank C
  - and so on …

Step (i): Interbank withdrawal behavior



- The run on Bank A causes it to withdraw from Bank D
- Bank D now has unusually high withdrawal demand, so it withdraws from Bank C
- Bank C then withdraws from Bank B …
- … causing Bank B to withdraw its deposit from Bank A

In other words

- A run on one bank ⇒ all interbank deposits withdrawn (**!**)

Step (ii): Fraction of investors served in Bank A:

- (Verify) q_A is the same as in the bilateral case

Step (iii): Payoffs of investors in Bank B

- a run on A necessarily spreads to B if:

$$c_{2,B}\left(e_A = 1 - \bar{\lambda}, e_B = e_C = e_D = 0; s_3\right) < c_1^* \qquad (1)$$

- (verify) exactly the same condition as in the bilateral case

- Assume (1) holds
  - if there is a run on Bank A, it necessarily spreads to Bank B
  - what is the implication for Banks C and D?

- If Bank B fails, we need to calculate the payout rate q_B
  - since Bank B is losing money on its deposit in Bank A …
  - can show: $q_B < q_A$ (Bank B is in worse shape than Bank A)
- Use q_B to calculate $c_{2,C}$ and ask if

$$c_{2,C}\left(e_A = e_B = 1 - \overline{\lambda}, e_C = e_D = 0; s_3\right) < c_1^* \qquad (2)$$

  - can show: if (1) holds, then (2) also holds
- In other words, if a run on A causes B to fail …
  - … then the run on B will cause C to fail …
  - … which will, in turn, cause D to fail (verify)

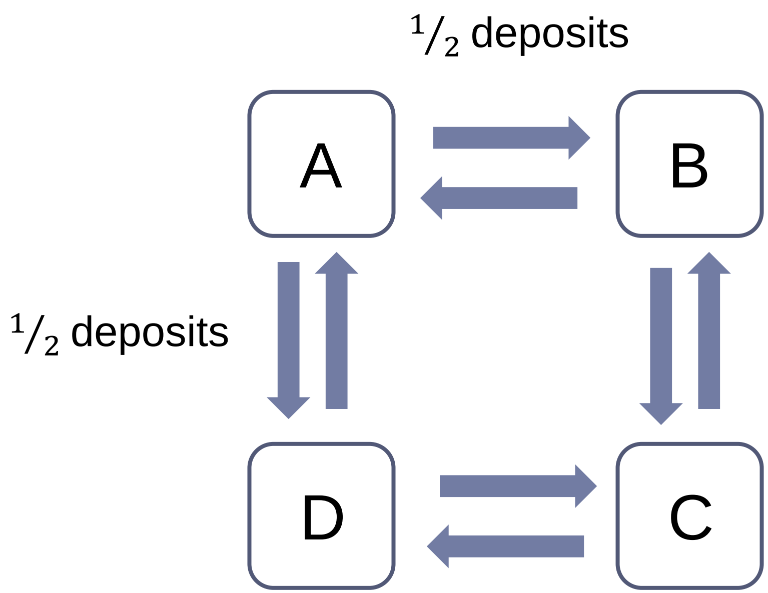
Result 4: With a circular network of interbank deposits

- a run is contagious under the same conditions as before
- but will now cause all banks to fail

- This is a striking result
  - Bank C had no (direct) dealing with Bank A
  - might have expected to be immune from A's problems
  - but ends up failing as part of a "domino effect"
- Small shocks can have very large consequences
  - imagine a circle network with 100+ banks
- Circle network is clearly more fragile than bilateral deposits

5.3 A complete network of deposits

Finally, suppose:

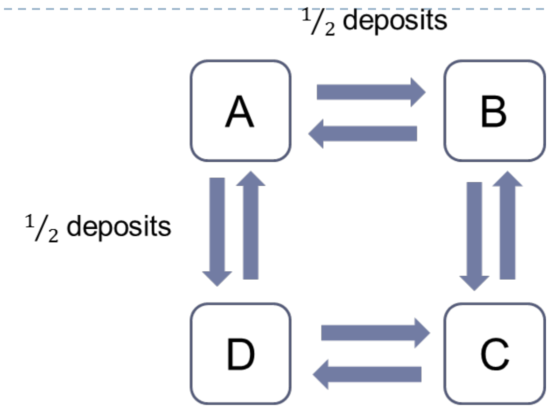


- There is again an equilibrium with

$$y_i^j = 2 \quad \forall\, i, \forall\, j$$

- What happens if there is a run on Bank A?

- Suppose $y_i^A = 1$ and
  $y_i^j = 2$ for $j = B, C, D$



- Follow the same steps:
  i. interbank withdrawal behavior
  ii. fraction of investors served in Bank A
  iii. payoffs of investors in Bank B (and Bank C)

focus again on state s_3

Step (i): Interbank withdrawal behavior

- run causes Bank A to withdraw from Banks B and D
- B and D now have high demand $\Rightarrow$ withdraw from A and C
- causing C to withdraw from B and D
- end result: all interbank deposits are withdrawn (again)

Step (ii): Fraction of investors served in Bank A:

- (Verify) q_A is the same as in the bilateral case

Step (iii): Payoffs of investors in Bank B (and Bank D)

- Bank B is better off than bilateral case
- because its deposit in Bank A was only half as large
- now must only liquidate $\frac{1}{2}\frac{(1-q_A)zc_1^*}{r}$ units of investment

- Calculate $c_{2,B}\left(e_A = 1-\bar{\lambda}, e_B = e_C = e_D = 0; s_3\right)$ as before
- Note: Bank D also suffers a loss on its interbank deposit

$$c_{2,D}(\cdot) = c_{2,B}(\cdot)$$

Result 5: If $c_{2,B}(e_A = 1 - \bar{\lambda}, e_B = e_C = e_D = 0; s_3) \geq c_1^*$

then y is a Nash equilibrium in state s_3.

- This condition is weaker than in the bilateral case
  - the run on Bank A is less likely to be contagious
  - in our example, it requires

$$\frac{9}{10}R \geq 1 \quad \text{or} \quad R \geq 1.11$$

Result 3: Otherwise, y is not a Nash equilibrium in s_3.

  - in this case, a run on Bank A necessarily causes a run on all other banks (verify)

- A run on Bank A is less likely to spread under a complete network than with bilateral deposits
  - the losses caused by A's failure are small for each bank
- But if it does spread, it causes all other banks to fail
  - whereas only Bank B fails in the bilateral case
- Illustrates an important tradeoff
  - is having more interbank exposures good or bad?
  - no easy answer – it depends on what type of shock hits
- Allen & Gale (2000) work through the implications of different network structures in more detail

6. Summary

Takeaways from Allen & Gale (2000)

- Interbank linkages are socially useful …
  - allow diversification of bank-specific liquidity risk
- …but make financial crises contagious
  - a trigger that causes a run on any one bank …
  - … could lead to the failure of many or all banks

  ⇒ small shocks can have very large consequences
- Strength of contagion depends on the size/pattern of these linkages
  - in practice this is <u>unknown</u> to policy makers
  - helps explain why predicting the course of events is difficult

- Example: the failure of Lehman Bros. in Sept. 2008
- Predicting the effects of this failure was very difficult
  - people recognized it would depend on interbank linkages
  - but "… understanding Lehman's current trading positions was tough. Lehman's roster of interest-rate swaps (a type of derivative investment) ran about two million [contracts]"
- One view: "because Lehman's troubles have been known for a while, … the market had had time to prepare."

  ⇒ govt. could allow Lehman to fail; effects would be contained
  - "We've re-established 'moral hazard' … Is that a good thing or a bad thing? We're about to find out."

    https://www.wsj.com/news/articles/SB122143670579134187

References

Franklin Allen and Douglas Gale (2007) *Understanding Financial Crises*, Oxford University Press.

- Chapter 10

Allen, Franklin and Douglas Gale (2000) "Financial Contagion," *Journal of Political Economy* 108: 1-33.

Extra Material

A Comment on Efficient Allocations When There is No Aggregate Uncertainty

- Consider a pure exchange economy with uncertainty
  - single time period
  - two states, s $= a, b$
- Two consumers, $i = 1,2$
- Strictly concave utility functions $u_i(c)$
- State-dependent endowments: $y_i(s)$
  - consumer 1: $(y_1(a), y_1(b)) = (3,1)$
  - consumer 1: $(y_2(a), y_2(b)) = (1,3)$

Q: What property must any Pareto optimal allocation satisfy?

A: $c_i(a) = c_i(b)$ for $i = 1,2$

- each consumers' consumption will be independent of the state

Why?

- Consider any allocation with $c_1(a) \neq c_1(b)$
  - then $c_2(a) \neq c_2(b)$
- The allocation $(\hat{c}_i(a), \hat{c}_i(b)) = \left(\frac{c_i(a)+c_i(b)}{2}, \frac{c_i(a)+c_i(b)}{2}\right)$
  - is feasible
  - is strictly preferred to c by both consumers
- This same property holds in the Allen-Gale model
  - uncertainty is about λ, the fraction of impatient investors, but …
  - no aggregate uncertainty implies that consumers should face no individual uncertainty in an efficient allocation

(return)

Deriving Properties of the Efficient Allocation

Setting up the planner's full problem

- To simplify notation, let's eliminate state s_3
  - set: $\text{prob}(s_1) = \text{prob}(s_2) = \frac{1}{2}$

- An allocation lists consumption plans in each location and each state:

$$\left\{\left(c_1^{i,j}(s), c_2^{i,j}(s)\right)\right\}_{i\in[0,1],\, j\in\{A,B\},\, s\in\{s_1,s_2\}}$$

- Again focus on symmetric allocations
  - investors in the same location are treated equally
  - plus: $c^A(s_1) = c^B(s_2)$ and $c^A(s_2) = c^B(s_1)$

- Recall: there is no *aggregate* uncertainty about λ
  - uncertainty is about where impatient investors will be located

- Some properties of any efficient allocation
  - no investment should be liquidated at $t = 1$
  - no storage should be held until $t = 2$

  } as before

- In our notation:

$$\lambda_H c_1^A(s_1) + \lambda_L c_1^B(s_1) = 1 - x$$
$$(1-\lambda_H) c_2^A(s_1) + (1-\lambda_L) c_2^B(s_1) = Rx$$

and

$$\lambda_L c_1^A(s_2) + \lambda_H c_1^B(s_2) = 1 - x$$
$$(1-\lambda_L) c_2^A(s_2) + (1-\lambda_H) c_2^B(s_2) = Rx$$

- Using symmetry, the first constraint becomes

$$\lambda_H c_1^A(s_1) + \lambda_L c_1^A(s_2) = 1 - x$$

note: we are **not** assuming $c_1^A(s_1) = c_1^B(s_1)$

Some first-order conditions

- The choice of (c_1^A, c_2^A) must maximize:

$$\frac{1}{2}\left(\lambda_H u\left(c_1^A(s_1)\right) + (1-\lambda_H) u\left(c_2^A(s_1)\right)\right) + \frac{1}{2}\left(\lambda_L u\left(c_1^A(s_2)\right) + (1-\lambda_L) u\left(c_2^A(s_2)\right)\right)$$

  - subject to $\lambda_H c_1^A(s_1) + \lambda_L c_1^A(s_2) = 1 - x$ and other constraints

- FOC for $c_1^A(s_1)$ and $c_1^A(s_2)$:

$$\frac{1}{2}\lambda_H u'\left(c_1^A(s_1)\right) = \lambda_H \mu$$

$$\frac{1}{2}\lambda_L u'\left(c_1^A(s_2)\right) = \lambda_L \mu$$

- Result: solution has $c_1^A(s_1) = c_1^A(s_2)$

The result

- The same steps can be applied to the planner's other choices

- Results:
  - $c_1^A(s) = c_1^A$ for all s and $c_2^A(s) = c_2^A$ for all s
  - $c_1^B(s) = c_1^B$ for all s and $c_2^B(s) = c_2^B$ for all s

- Symmetry now implies: $c_1^A = c_1^B$ and $c_2^A = c_2^B$

Result:

- Any efficient allocation is completely characterized by two numbers: (c_1, c_2)

(return)