Supplemental appendix to
"Liquidity regulation and the implementation of monetary policy"

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Abstract

In this appendix, we do four things: (A) provide a proof of Proposition 1; (B) present an extension of the model in which the LCR requirement lessens the impact of a liquidity crisis; (C) show that the effects of liquidity regulation are amplified when the runoff rate for loans from the central bank, θ_X, is larger than the runoff rate for deposits, θ_D; and (D) show that the effects of liquidity regulation are also amplified when the central bank uses repurchase agreements rather than outright transactions.

A. Proof of Proposition 1

Proposition 1 *Bank i will choose $\{\Delta_t^i\}$ so that (E_R^i, E_{LCR}^i) satisfy*

$$r_t = r_R + (r_X - r_R)\begin{pmatrix} 1 - G\left[\max\left\{E_R^i, \hat{\varepsilon}^i\right\}\right] \\ +\frac{1-\theta_t}{1-\theta_X}\max\left\{G\left[\hat{\varepsilon}^i\right] - G\left[E_{LCR}^i\right], 0\right\} \end{pmatrix} \tag{13}$$

for $t = 1, 2$.

Proof: Given that density function of the payment shock ε^i is continuous, it is straightforward to show that the objective function in (12) is continuously differentiable for all $\{\Delta_t^i\} \in \mathbb{R}^2$. The fact that the support of ε^i is bounded ensures that a solution to the problem exists for any market interest rates satisfying $r_2 \geq r_1 \geq r_R$. To deal with the indicator function in the objective, we examine the first-order necessary conditions in the regions where $E_R^i < E_{LCR}^i$ and where $E_R^i > E_{LCR}^i$ separately. (See the two panels of Figure 4.) If $E_R^i < E_{LCR}^i$, the indicator function is zero and we have

$\max\left\{E_R^i, \hat{\varepsilon}^i\right\} = E_R^i$. In this region, the marginal net gain of borrowing in each market is given by

$$\frac{\partial E[\pi^i]}{\partial \Delta_t^i} = -r_t + r_R + (r_X - r_R)\int_{E_R^i}^{\infty} dG\left(\varepsilon^i\right). \tag{24}$$

In the region where $E_R^i > E_{LCR}^i$ holds, the value of the indicator function is one, we have $\max\left\{E_R^i, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$, and the marginal net gains are given by[18]

$$\frac{\partial E[\pi^i]}{\partial \Delta_t^i} = -r_t + r_R + (r_X - r_R)\left(\frac{1-\theta_t}{1-\theta_X}\int_{E_{LCR}^i}^{\hat{\varepsilon}^i} dG\left(\varepsilon^i\right) + \int_{\hat{\varepsilon}^i}^{\infty} dG\left(\varepsilon^i\right)\right). \tag{25}$$

We divide the proof into four cases, depending on the configuration of market interest rates.

(a) If $r_2 = r_1 = r_R$, (24) and (25) show that bank i's profit will be strictly increasing in Δ_1^i and/or Δ_2^i unless the final term in each equation – which is proportional to the probability that the bank will need to borrow from the central bank at the end of the period – is zero. Any plan $\left\{\Delta_t^i\right\}$ that solves the problem must, therefore, ensure that $G\left[E_R^i\right] = G\left[E_{LCR}^i\right] = 1$, and the bank will be indifferent between all such plans. It is straightforward to see that these conditions are equivalent to the solution given in (13) for this case.

(b) If $r_2 = r_1 > r_R$, (25) implies that a solution cannot satisfy $E_R^i > E_{LCR}^i$ because the bank can increase its expected profit anywhere in this region by lending one more unit overnight and either borrowing one more unit at term (if $G\left[E_{LCR}^i\right] < 1$) or holding one unit less of excess reserves (if $G\left[E_{LCR}^i\right] = 1$). Any solution must, therefore, lie in the region where $E_R^i \leq E_{LCR}^i$ holds. Using the fact that $\max\left\{E_R^i, \hat{\varepsilon}^i\right\} = E_R^i$ in this region, it is straightforward to show that setting the first-order condition in (24) to zero for $t = 1, 2$ is equivalent to the the pricing equations in (13) for this case. There are again many plans $\left\{\Delta_t^i\right\}$ that solve the problem, but all of these plans generate the same value for E_R^i and ensure that $E_R^i \leq E_{LCR}^i$ holds.

(c) If $r_2 > r_1 = r_R$, (24) implies that a solution cannot satisfy $E_R^i \leq E_{LCR}^i$ because the bank can increase its expected profit anywhere in this region by lending one more unit at term and borrowing one more unit overnight. A solution must, therefore, lie in the region where $E_R^i > E_{LCR}^i$ holds. Moreover, (25) implies that expected profit is strictly increasing in Δ_1^i is this case unless $G\left[\hat{\varepsilon}^i\right] = 1$. The pricing equations in (13) for this case follow from using $G\left[\hat{\varepsilon}^i\right] = 1$ together with the fact that $\max\left\{E_R^i, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$ in this region and setting (25) to zero for $t = 2$. Condition (25) generates a

[18] Note that when $\hat{E}_R^i = \hat{E}_{LCR}^i$ holds, the derivatives in (24) and (25) are equal. Either expression can be used to evaluate the net gain of borrowing along this border between the two regions.

unique solution for Δ_2^i (and hence for E_{LCR}^i) in this case. There are many solutions for Δ_1^i, but all of them leave the bank holding enough excess reserves that $G\left[\hat{\varepsilon}^i\right] = 1$ holds.

(d) If $r_2 > r_1 > r_R$, then as in case (c), (24) implies that a solution cannot satisfy $E_R^i \leq E_{LCR}^i$ because the bank can increase its expected profit in this region by lending more at term and borrowing the same amount overnight. Any solution must, therefore, lie in the region where $E_R^i > E_{LCR}^i$ holds and we have $\max\left\{E_R^i, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$. Setting the first-order condition (25) to zero for $t = 1, 2$ yields a unique solution for $\left\{\Delta_t^i\right\}$ in this case and is easily seen to be equivalent to the pricing equation in (13). ■

B. The LCR and liquidity crises

The model and analysis in the main text focus on potential side effects of the LCR requirement while abstracting from the benefits of the new regulation. In this section, we show how some of these benefits can be captured within the same basic framework by extending the model to include an additional period and assuming that one or more banks may lose access to interbank borrowing in this period.

Specifically, we extend the timeline in Figure 2 to include a fourth stage representing the next trading day. At the beginning of this new stage, banks repay their overnight borrowing from other banks and any borrowing from the central bank. In addition, the payment shocks ε^i are reversed, so that bank i's deposits return to their original level D^i. The bank's balance sheet does not fully return to the initial situation in Figure 1, however, because the term interbank loans Δ_2^i remain in place, as shown in Figure 10.

Bank i

Assets		Liabilities	
Loans	L^i	Deposits	D^i
Bonds	B^i	Net interbank borrowing	Δ_2^i
Reserves	$R^i + \Delta_2^i$	Equity	Y^i

Figure 10: Balance sheet of bank i at the beginning of stage IV

At this point, one or more banks may experience an event we call a *liquidity crisis.* In this event, the affected banks lose access to new interbank borrowing while, at the same time, experiencing an extraordinary outflow of η deposits. Define the *funding gap* of affected bank i to be the amount of this outflow that it cannot meet using its liquid assets (both reserves and bonds), that is,

$$F^i(\eta) \equiv \max\left\{\eta - (R^i + \Delta_2^i + B^i), 0\right\}. \tag{26}$$

In other words, the funding gap represents the amount of money the bank must raise by either borrowing from the central bank or selling illiquid assets. This gap depends on bank i's liquid asset holdings and may differ across banks. Define the *expected funding gap* to be the gap faced by a randomly-chosen bank,

$$F(\eta) \equiv \mathbb{E}_i\left[F^i(\eta)\right] = \int_0^1 \max\left\{\eta - R^i - \Delta_2^i - B^i, 0\right\} di. \tag{27}$$

We are interested in how the introduction of an LCR requirement affects the expected funding gap during a liquidity crisis.

In the absence of an LCR requirement, the term market is redundant in our model and we can set $\Delta_2^i = 0$ for all i. To simplify the calculations, assume all banks have the same level of deposits (that is, $D^i = D$ for all i) and define λ^i to be the initial ratio of bank i's liquid assets to its deposits, that is,

$$\lambda^i \equiv \frac{R^i + B^i}{D}.$$

Suppose that λ^i is distributed across banks according to some continuous distribution function H with full support on $[0,1]$ and with density h. We can then write the expected funding gap when there is no LCR requirement as

$$\begin{aligned} F_N(\eta) &= \int_0^1 \max\{\eta - \lambda D, 0\}\, h(\lambda)\, d\lambda \\ &= \int_0^{\frac{\eta}{D}} (\eta - \lambda D)\, h(\lambda)\, d\lambda. \end{aligned} \tag{28}$$

It is straightforward to show that $F_N(0) = 0$ and that $F_N(D) = \left(1 - \bar{\lambda}\right) D$, where $\bar{\lambda}$ is the average value of λ^i across banks. Note that this average is equal to the ratio of liquid assets to deposits for the banking system as a whole,

$$\bar{\lambda} = \frac{R + B}{D}. \tag{29}$$

It is also easy to see that the function F_N is strictly increasing in η.

When a bank that has lost access to interbank borrowing must borrow from the central bank and/or sell illiquid assets, these actions may generate externalities. For this reason, policy makers may want to impose regulations that encourage banks to better position themselves to withstand such an event. One stated goal of the LCR is "to prevent central banks from becoming the 'lender of first resort"' in a liquidity crisis (BCBS, 2013a). In our framework, the LCR requirement serves precisely this purpose by reducing the expected funding gap F. To simplify the calculations, assume $\theta_2 = 0$, meaning that term borrowing counts fully toward the LCR requirement. Then bank i's equilibrium borrowing in the term market is given by

$$\begin{aligned}\Delta_2^i &= (1-\theta_D)\left(E_{LCR} - E_{LCR}^i\right) \\ &= B + R - B^i - R^i.\end{aligned}$$

Substituting this expression into (27) yields an expression for the funding gap of bank i that is independent of the bank's initial balance sheet,

$$F_{LCR}^i(\eta) = \max(\eta - R - B, 0). \tag{30}$$

With the LCR requirement in place, trade in interbank markets leads to an even distribution of excess LCR liquidity across all banks. As a result, the funding gap faced by an individual bank will depend on the quantity of liquid assets in the banking system rather than on its own holding of liquid assets before the market opens. This fact implies that the expected funding gap is also given by (30) when the LCR requirement in place. Using (29), we can rewrite this expression as

$$F_{LCR}(\eta) = \max\left\{\eta - \bar{\lambda}D, 0\right\}. \tag{31}$$

It is easy to see that F_{LCR} has the same starting and ending points as F_N, that is, $F_{LCR}(0) = 0$ and $F_{LCR}(D) = \left(1-\bar{\lambda}\right)D$. The following proposition is the main result of this section: for all interior values of η, the expected funding gap associated with a liquidity crisis is smaller when the LCR requirement is in place.

Proposition 5 *The introduction of an LCR requirement strictly decreases the expected funding gap, that is,* $F_{LCR}(\eta) < F_N(\eta)$*, for any* $\eta \in (0, D)$*.*

The proof of this proposition is a straightforward application of Jensen's inequality to (28) and (31), using the full support assumption for the distribution H and the fact that the max operator generates a convex function of λ.

Figure 11 illustrates this result for the case where the distribution H is uniform. The solid curve represents the expected funding gap with no LCR requirement, F_N. As expected, this curve is strictly increasing in η. With an LCR requirement, the expected funding gap is given by the dashed curve. All banks are now able to meet a modest outflow (here, any $\eta \leq D/2$) using their own bonds and reserves and, therefore, the expected funding gap is zero in this range. If the deposit outflow is larger than $D/2$, an affected bank will exhaust its liquid assets and be forced to borrow from the central bank and/or sell illiquid assets. However, as established in Proposition 5, the average amount of this activity across banks will always be less than in an environment without an LCR requirement. In this sense, the LCR requirement can be seen as promoting financial stability in the extended version of our model.

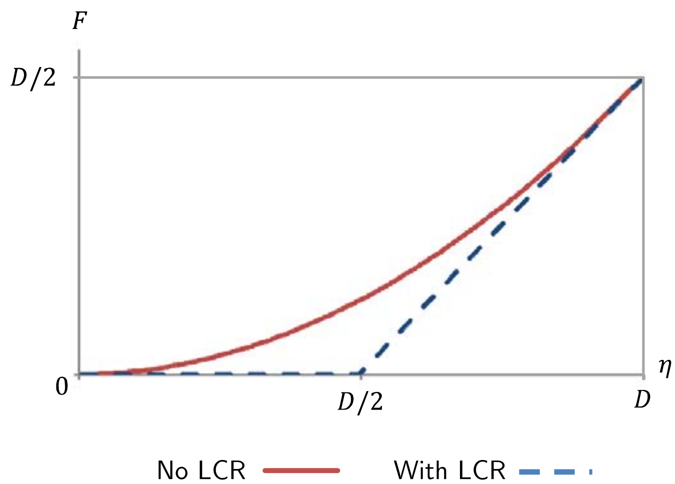

Figure 11: Expected funding gap in a liquidity crisis

C. The model with $\theta_X > \theta_D$

The original LCR rules published in 2010 required that the runoff rate θ_X for loans from the central bank be set to at least 25%, but this minimum has since been lowered to 0% (BCBS, 2013b). As a result, local jurisdictions have the freedom to set θ_X to any value they choose. The analysis in the text assumes that the runoff rate on loans from the central bank, θ_X, is set below the runoff rate on deposits, θ_D. In this section, we extend the analysis to the case where $\theta_X > \theta_D$ holds and show how the effects of the LCR regulation on equilibrium interest rates are amplified in this case. In addition, we show that the equilibrium LCR premium is (weakly) increasing in θ_X over the entire range.

The demand for interbank loans. When $\theta_X > \theta_D$ holds, the amount bank i must borrow from the central bank's lending facility to meet all of its regulatory requirements is still given by

(8), but the pattern of borrowing depicted in Figure 4 in the main text changes to that in Figure 12. The principal difference is that the (blue) line associated with borrowing for LCR purposes is now steeper than the (green) line associated with the reserve requirement. As a result, the LCR requirement now determines the amount a bank must borrow from the central bank for sufficiently large values of the payment shock in panel (i) and for all values of the payment shock in panel (ii).

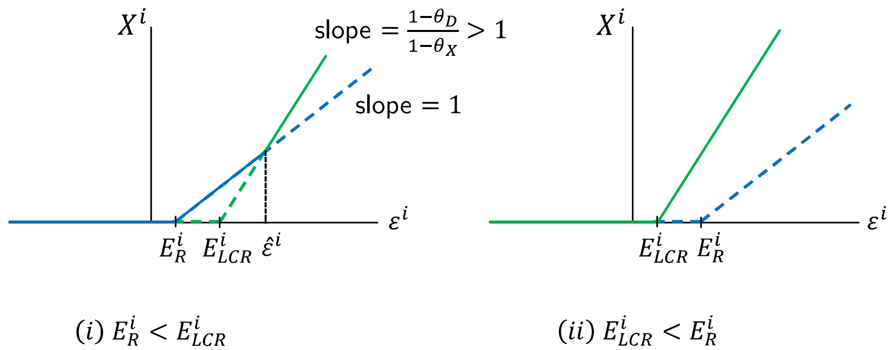

Figure 12: Bank i's borrowing from the central bank (with $\theta_X > \theta_D$)

The objective function bank i seeks to maximize when choosing its interbank borrowing activity $\{\Delta_t^i\}$ changes from (12) to

$$-\sum_t r_t\Delta_t^i + r_R E_R^i - (r_X - r_R)\left\{\begin{array}{c}\mathbb{I}_{(E_R^i \le E_{LCR}^i)}\int_{E_R^i}^{\hat{\varepsilon}^i}\left(\varepsilon^i - E_R^i\right)dG\left(\varepsilon^i\right)\\ +\int_{\max\{E_{LCR}^i,\hat{\varepsilon}^i\}}^{\infty}\frac{1-\theta_D}{1-\theta_X}\left(\varepsilon^i - E_{LCR}^i\right)dG\left(\varepsilon^i\right)\end{array}\right\}.$$

The solution to this problem is characterized in the following proposition.

Proposition 6 *When $\theta_X > \theta_D$, bank i will choose $\{\Delta_t^i\}$ so that $\left(E_R^i, E_{LCR}^i\right)$ satisfy*

$$r_t = r_R + (r_X - r_R)\left(\frac{1-\theta_t}{1-\theta_X}\left(1 - G\left[\max\left\{E_{LCR}^i, \hat{\varepsilon}^i\right\}\right]\right) + \max\left\{G\left[\hat{\varepsilon}^i\right] - G\left[E_R^i\right], 0\right\}\right) \tag{32}$$

for $t = 1, 2$, where $\hat{\varepsilon}^i$ is as defined in (9).

Proof: The general structure of the proof follows that of Proposition 1. It is again straightforward to show that the objective function is continuously differentiable for all $\{\Delta_t^i\} \in \mathbb{R}^2$ and that a solution to the problem exists for any market interest rates satisfying $r_2 \geq r_1 \geq r_R$. To deal with the indicator function in the objective, we proceed as before by examining the first-order conditions in the regions where $E_R^i < E_{LCR}^i$ and where $E_R^i > E_{LCR}^i$ separately. If $E_R^i < E_{LCR}^i$, the indicator function is one and we have $\max\left\{E_{LCR}^i, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$. In this region, the marginal net gain of borrowing in each market is given by

$$\frac{\partial E[\pi^i]}{\partial \Delta_t^i} = -r_t + r_R + (r_X - r_R)\left(\int_{E_R^i}^{\hat{\varepsilon}^i} dG\left(\varepsilon^i\right) + \frac{1-\theta_t}{1-\theta_X}\int_{\hat{\varepsilon}^i}^{\infty} dG\left(\varepsilon^i\right)\right). \tag{33}$$

In the region where $E_R^i > E_{LCR}^i$, the value of the indicator function is zero, we have $\max\left\{E_{LCR}^i, \hat{\varepsilon}^i\right\} = E_{LCR}^i$, and the marginal net gains are given by[19]

$$\frac{\partial E[\pi^i]}{\partial \Delta_t^i} = -r_t + r_R + (r_X - r_R)\frac{1-\theta_t}{1-\theta_X}\int_{E_{LCR}^i}^{\infty} dG\left(\varepsilon^i\right). \tag{34}$$

We again divide the proof into four cases, depending on the configuration of market interest rates.

(a) If $r_2 = r_1 = r_R$, (33) and (34) show that bank i's profit will be strictly increasing in Δ_1^i and/or Δ_2^i unless the final term in each equation is zero. Any plan $\left\{\Delta_t^i\right\}$ that solves the problem must, therefore, ensure that $G\left[E_R^i\right] = G\left[E_{LCR}^i\right] = 1$ holds, and the bank will be indifferent between any such plans. It is straightforward to see that these two conditions are equivalent to the solution given in (32) for this case.

(b) If $r_2 = r_1 > r_R$, using (34) for $t = 1$ shows that a solution cannot satisfy $E_R^i \geq E_{LCR}^i$ because the bank can increase its expected profit anywhere in this region by increasing its overnight lending and holding fewer excess reserves. Any solution must, therefore, lie in the region where $E_R^i < E_{LCR}^i$. Moreover, (33) shows that the bank can increase its expected profit in this region by borrowing at term and lending the funds overnight unless $G\left[\hat{\varepsilon}^i\right] = 1$ holds. The pricing equations in (32) for this case follow from using $G\left[\hat{\varepsilon}^i\right] = 1$ together with the fact that $\max\left\{E_{LCR}^i, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$ in this region and setting (33) to zero for $t = 1$. There are again many plans $\left\{\Delta_t^i\right\}$ that solve the problem, but all of them generate the same value for E_R^i and ensure the bank has enough excess LCR liquidity that $G\left[\hat{\varepsilon}^i\right] = 1$ holds.

(c) If $r_2 > r_1 = r_R$, (33) implies that a solution cannot satisfy $E_R^i < E_{LCR}^i$ because the bank can increase its expected profit anywhere in this region by borrowing one more unit overnight and either holding one more unit of excess reserves (if $G\left[E_R^i\right] < 1$) or lending one more unit at term (if $G\left[E_R^i\right] = 1$). A solution must, therefore, lie in the region where $E_R^i \geq E_{LCR}^i$ holds. It is straightforward to show that setting the first-order condition in (34) to zero and using the fact that $\max\left\{E_{LCR}^i, \hat{\varepsilon}^i\right\} = E_{LCR}^i$ in this region yields the pricing equation in (32) for $t = 1, 2$. Condition (34) generates a unique solution for Δ_2^i in this case. There are again many solutions for Δ_1^i, but

[19] These two expressions for the derivatives are again equal along the boundary where $E_R^i = E_{LCR}^i$.

all of them leave the bank holding enough excess reserves that $E^i_R \geq E^i_{LCR}$ holds.

(d) If $r_2 > r_1 > r_R$, then as in case (b), (34) implies that a solution cannot satisfy $E^i_R \geq E^i_{LCR}$ because the bank can increase its expected profit in this region by lending more overnight and holding fewer excess reserves. The solution must, therefore, lie in the region where $E^i_R < E^i_{LCR}$ holds and we have $\max\left\{E^i_{LCR}, \hat{\varepsilon}^i\right\} = \hat{\varepsilon}^i$. Setting the first-order condition (33) to zero for $t = 1, 2$ yields a unique solution for $\left\{\Delta^i_t\right\}$ in this case and is easily seen to be equivalent to the pricing equation in (32). ■

As in the main text, we can, without any loss of generality focus on symmetric outcomes in which banks trade to the same levels of excess reserves E^i_R and excess LCR liquidity E^i_{LCR} in cases where they are indifferent about one or both of these levels.

Equilibrium interest rates. As in the main text, the market-clearing conditions, $\Delta_t = 0$ for $t = 1, 2$, can be written as (17) and (18). Substituting these expressions into the demand functions from Proposition 6 and using the definition of $\hat{\varepsilon}$ in (19) yields the equilibrium pricing relationships for this case.

Proposition 7 *When $\theta_X > \theta_D$, the unique equilibrium interest rate in market $t = 1, 2$ is given by*

$$r^*_t = r_R + (r_X - r_R)\left(\frac{1-\theta_t}{1-\theta_X}\left(1 - G\left[\max\left\{E_{LCR}, \hat{\varepsilon}\right\}\right]\right) + \max\{G[\hat{\varepsilon}] - G[E_R], 0\}\right).$$

This result provides the counterpart to Proposition 2 in the main text. Next, we use this result to establish that the equilibrium LCR premium is an increasing function of the runoff rate θ_X for loans from the central bank. The following proposition spans both the case of $\theta_X < \theta_D$ studied in the main text and that of $\theta_X > \theta_D$ studied here.

Proposition 8 *The equilibrium LCR premium $r^*_2 - r^*_1$ is a (weakly) increasing function of θ_X.*

Proof: To begin, note that $\hat{\varepsilon}$ defined in (19) is a continuous function of θ_X in the regions $\theta_X < \theta_D$ and $\theta_X > \theta_D$ separately, but is discontinuous at $\theta_X = \theta_D$. Differentiating yields

$$\frac{\partial \hat{\varepsilon}}{\partial \theta_X} = \frac{(1-\theta_D)(E_R - E_{LCR})}{(\theta_D - \theta_X)^2} \tag{35}$$

for any $\theta_X \neq \theta_D$. The remainder of the proof is divided into three parts.

Part (i) : $\theta_X < \theta_D$. In this region, we can use Proposition 2 to write the equilibrium regulatory premium as

$$r_2^* - r_1^* = (r_X - r_R)\left(\frac{1-\theta_2}{1-\theta_X}\max\left\{G\left[\hat{\varepsilon}\right] - G\left[E_{LCR}\right], 0\right\}\right). \tag{36}$$

Since $\hat{\varepsilon}$ is a continuous function of θ_X in this region, it follows that the LCR premium is a continuous function of θ_X in this region. To see that it is also a (weakly) increasing function, first note that if $\hat{\varepsilon} \leq E_{LCR}$, then the premium is equal to zero for any θ_X. When $\hat{\varepsilon} > E_{LCR}$, one can use (19) and (35) to show that the following inequalities are equivalent:

$$\hat{\varepsilon} > E_{LCR} \quad \Leftrightarrow \quad E_R > E_{LCR} \quad \Leftrightarrow \quad \frac{\partial \hat{\varepsilon}}{\partial \theta_X} > 0. \tag{37}$$

(These relationships can also be seen in Figure 4, where the slope of the flatter (green) line is increasing in θ_X.) Since the distribution function G is (weakly) increasing, it follows that the expression for the premium in (36) is increasing in θ_X whenever $\hat{\varepsilon} > E_{LCR}$. We have, therefore, shown that the LCR premium is an increasing of θ_X for all $\theta_X < \theta_D$.

Part (ii) : $\theta_X > \theta_D$. In this region, we can use Proposition 7 to write the premium as

$$r_2^* - r_1^* = (r_X - r_R)\frac{1-\theta_2}{1-\theta_X}\left(1 - G\left[\max\left\{E_{LCR}, \hat{\varepsilon}\right\}\right]\right). \tag{38}$$

If $\hat{\varepsilon} \leq E_{LCR}$, the argument of G is independent of θ_X and the overall expression is clearly increasing in θ_X (and strictly increasing if $G\left[E_{LCR}\right] < 1$). For the reverse case, using (19) and (35) shows that the following inequalities are equivalent when $\theta_X > \theta_D$:

$$\hat{\varepsilon} > E_{LCR} \quad \Leftrightarrow \quad E_R < E_{LCR} \quad \Leftrightarrow \quad \frac{\partial \hat{\varepsilon}}{\partial \theta_X} < 0. \tag{39}$$

(See also Figure 12, where the slope of the steeper (green) line is increasing in θ_X.) Since the distribution function G is (weakly) increasing and bounded above by 1, it follows that the expression for the premium in (38) is increasing in θ_X in the region where $\hat{\varepsilon} > E_{LCR}$ as well. This establishes that the LCR premium is an increasing function of θ_X for all $\theta_X > \theta_D$.

Part (iii): $\theta_X = \theta_D$. What remains is to show that the equilibrium LCR premium does not decrease as θ_X crosses θ_D, the boundary that separates the two regions discussed above. We divide this analysis into three cases. First, suppose that $E_R < E_{LCR}$ holds. In this case, it follows from (36) and the discussion above that the LCR premium is zero for all $\theta_X < \theta_D$. For the region where

$\theta_X > \theta_D$, the definition of $\hat{\varepsilon}$ in (19) implies

$$\lim_{\theta_X \downarrow \theta_D} \hat{\varepsilon}\left(\theta_x\right) = \infty.$$

(This fact can also be seen in panel (i) of Figure 12, where the slope of the steeper (green) line falls to one as θ_x decreases to θ_D.) Using the expression for the premium in (38), we then have

$$\lim_{\theta_X \downarrow \theta_D} r_2^* - r_1^* = \left(r_X - r_R\right) \frac{1-\theta_2}{1-\theta_X} \left(1 - G\left[\infty\right]\right) = 0.$$

This equation establishes that the regulatory premium increases from zero as θ_X rises above θ_D and, hence, is a continuous function of θ_X when $E_R \leq E_{LCR}$ holds.

Now suppose $E_R > E_{LCR}$ holds. The expression for $\hat{\varepsilon}$ in (19) now implies

$$\lim_{\theta_X \uparrow \theta_D} \hat{\varepsilon}\left(\theta_x\right) = \infty$$

(see also panel (ii) of Figure 4, where the slope of the flatter (green) line rises to one as θ_X increases to θ_D). Using this fact and (36), the limiting value of the premium as θ_X approaches θ_D from below is given by

$$\lim_{\theta_X \uparrow \theta_D} r_2^* - r_1^* = \left(r_X - r_R\right) \frac{1-\theta_2}{1-\theta_X} \left(1 - G\left[E_{LCR}\right]\right).$$

In the region where $\theta_X > \theta_D$, the inequalities in (39) show that $\hat{\varepsilon} < E_{LCR}$ holds in this case and, using (38), the limiting value of the premium as θ_X approaches θ_D from above is given by

$$\lim_{\theta_X \downarrow \theta_D} r_2^* - r_1^* = \left(r_X - r_R\right) \frac{1-\theta_2}{1-\theta_X} \left(1 - G\left[E_{LCR}\right]\right).$$

The equality of these last two expressions shows that the equilibrium regulatory premium is also a continuous function of θ_X when $E_R > E_{LCR}$ holds. The third and final case is where $E_R = E_{LCR}$. In this case, the value of $\hat{\varepsilon}$ may jump discontinuously as θ_X crosses θ_D,

$$\lim_{\theta_X \uparrow \theta_D} r_2^* - r_1^* = 0 \leq \left(r_X - r_R\right) \frac{1-\theta_2}{1-\theta_D} \left(1 - G\left[E_{LCR}\right]\right) = \lim_{\theta_X \downarrow \theta_D} r_2^* - r_1^*.$$

Because this jump (which occurs if $G\left[E_{LCR}\right] < 1$) is upward, we have established that the equilibrium LCR premium is everywhere increasing in θ_X, as desired. ■

Open Market operations with $\theta_X > \theta_D$. As a final step in this section of the Appendix, we show how the impact of introducing an LCR requirement on the outcome of a central bank's open market operations is amplified when $\theta_X > \theta_D$ holds. The effect of an open market operation on the balance sheet of the banking system is again as depicted in Figure 6 in the main text. The relationship between these changes and equilibrium interest rates, however, is now governed by Proposition 7. Figure 13 illustrates the results for this case by plotting the equilibrium interest rates $\{r_t^*\}$ associated with two different values of θ_X in each panel: $\theta_X = 0.25$ and $\theta_X = 0$. These runoff rates correspond to the minimum standards under the original and revised LCR rules, respectively.[20] All other parameter values are the same as in Figure 8 in the main text, and the dashed curves in Figure 13 are the same curves as in the first two panels of Figure 8.

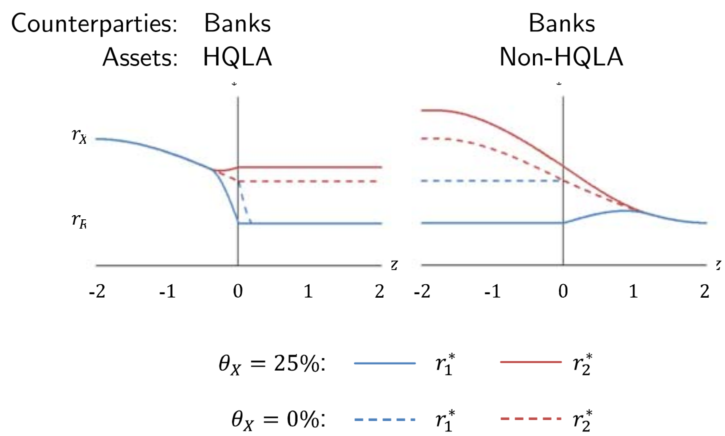

Figure 13: Effects of an OMO with $\theta_X > \theta_D$ vs. $\theta_X = 0$

For the case of operations with HQLA, when z is sufficiently negative, the LCR requirement is slack and equilibrium interest rates are the same as in the standard model regardless of the value of θ_X. For sufficiently positive values of z, the overnight rate is at the floor of the corridor for both values of θ_X. For intermediate values of z, however, we see that the higher value of θ_X pushes the overnight rate lower. The difference can be significant: when $z = 0$, the overnight rate is at the midpoint of the corridor with $\theta_X = 0$ but at the floor of the corridor with $\theta_X = 0.25$. In addition, the higher value of θ_X pushes the term rate up substantially for both moderate and sufficiently positive values of z. Overall, the left panel of Figure 13 shows how a higher runoff rate θ_X tends to increase the regulatory premium and amplify the effect of the LCR requirement on equilibrium interest rates.

When the central bank conducts operations using non-HQLA, the same general effects are present but the differences between the two cases are even larger. With $\theta_X = 0$, the overnight rate

[20] The original LCR standards published in 2010 are available at http://www.bis.org/publ/bcbs188.pdf. The revised standards published in 2013 are available at http://www.bis.org/publ/bcbs238.pdf.

lies at the midpoint of the corridor for all $z < 0$. With $\theta_X = 0.25$, in contrast, the overnight rate lies on the floor of the corridor for all $z < 0$. In addition, the term interest rate can now rise above the ceiling of the corridor, r_X, since a dollar of term borrowing in this case will save the bank from having to borrow $(1 - \theta_X)^{-1} > 1$ dollars from the central bank's lending facility in some states.

Together, the two panels in Figure 13 clearly illustrate how the effects of liquidity regulation on equilibrium interest rates identified in the main text of the paper for the case of $\theta_X < \theta_D$ are amplified when θ_X is instead set larger than θ_D.

D. Repurchase agreements

In this section, we show how the effects of an open market operation change when it is structured as a repurchase agreement (repo) rather than as an outright purchase/sale by the central bank. In a repo transaction, the exchange of reserves for assets between the central bank and its counterparties is reversed at a pre-specified future date. The key difference between a repo and an outright transaction in our framework is that a repo typically involves a *haircut*, meaning that the value of the assets encumbered by the operation is larger than the quantity of reserves created. We show here that this fact tends to amplify the equilibrium LCR premium when the central bank adds reserves in an operation using HQLA as countervalue and banks as counterparties.[21]

Figure 14 shows the balance sheet effects of an operation in which the central bank purchases z_B units of bonds using a repurchase agreement with haircut $h \geq 0$ and a fraction α_B of these bonds are sold by banks. Taking into account the haircut, the total quantity of bonds in the banking system that are encumbered by this operation is $\alpha_B z_B / (1 - h)$. As the LCR rules only allow unencumbered assets to be included in the calculation of a bank's HQLA, an operation with $\alpha_B = 1$ now decreases the LCR position of the banking system rather than leaving it unchanged.[22]

[21] The results in this section also apply to collateralized loans, which in the context of our model are identical to a repo transaction with the same haircut. In practice, repurchase agreements generally require that the collateral involved is marketable, while collateralized loans can be used with a much wider range of countervalue.

[22] It is straightforward to see that the effects of a repo operation using only loans ($z_B = 0$) or with only non-bank counterparties ($\alpha_B = 0$) are exactly the same as the outright transactions studied in the main text. While a repo transaction again encumbers additional assets, Propositions 2 and 7 show that the quantities of unencumbered loans held by banks and unencumbered bonds held by non-banks have no effect on equilibrium interest rates in our model.

Central Bank

Assets		Liabilities	
Loans	L_0^{CB}	Reserves	$R_0^{CB} + z_B$
Bonds	$B_0^{CB} + z_B$	Equity	Y^{CB}

Banking System

Assets		Liabilities	
Loans	L_0	Deposits	$D_0 + (1 - \alpha_B)z_B$
Bonds	B_0		
- encumbered	$a_B z_B/(1-h)$		
Reserves	$R_0 + z_B$	Equity	Y

Figure 14: Balance sheet effects of a repo operation

Figure 15 presents the effects of an open market operation structured as a repurchase agreement on the equilibrium interest rates in our model. The parameter values are the same as those used to generate Figure 8 in the main text. The dashed curves in the figure show the effects of an operation with zero haircut, which is equivalent to the outright purchase/sale depicted in the left panel of Figure 8. The solid lines show the equilibrium interest rates when the haircut is set to 25%. (This relatively large value for the haircut is useful for illustrating the effects visually in the graph.)

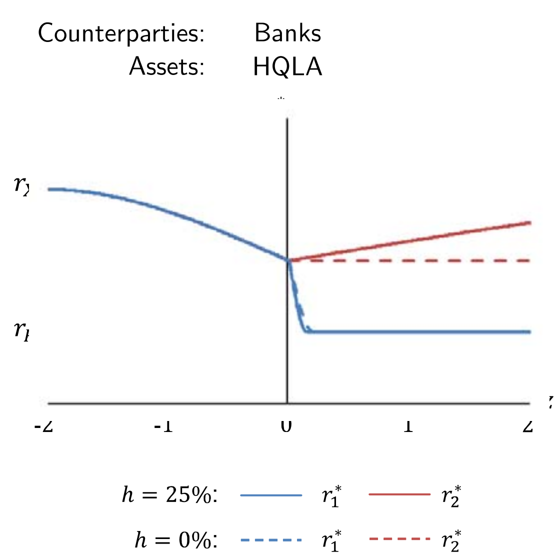

Figure 15: Effects of a repo operation with banks using HQLA

As the figure shows, the regulatory premium is larger when the haircut is higher and the difference increases with the size of the operation. In particular, the higher haircut causes the overnight interest rate to fall slightly faster as reserve supply increases, while it causes the term interest rate to rise. The fact that the term interest rate increases when the central bank adds reserves is particularly striking; this result can be understood by looking back at Figure 4. When the central bank adds reserves using this operation, banks' excess reserves E_R move to the right as before. However, the quantity of unencumbered high-quality liquid assets held by the banking system now falls due to the haircut, which moves excess LCR liquidity E_{LCR} to the left in the figure. The result is that a bank's need to borrow from the central bank will be determined by the LCR requirement for a wider range of values of the payment shock ε^i, which drives up the interest rate on term loans and widens the regulatory premium, as seen in Figure 15. The figure thus illustrates how the effects studied in the main text are amplified when the central bank uses repurchase agreements.