Run Equilibria in the Green-Lin Model of Financial Intermediation

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Introduction

• Financial intermediaries are commonly believed to be inherently “fragile”

• Take short-term deposits, make long-term investments

• Result: illiquidity
  • short-term liabilities > short-term assets

• If all investors withdraw funds at once, intermediary will fail
  • if intermediary will fail, investors want to withdraw
    ⇒ hints at possibility of a self-fulfilling bank run

• Classic model: Diamond & Dybvig (1983)
• Maturity transformation/illiquidity is not limited to banks
  • also performed by other financial institutions and in markets

• Examples:
  • Asset-backed commercial paper
  • Money-market/cash management funds
  • Auction-rate securities
  • Investment banks (Bear Stearns, Lehman Bros.)

• Many recent events appear “similar” to a bank run
  • Eichengreen: “What happened to Bear Stearns ... looked a lot like a 19th century run on the bank.”
• Want to be able to evaluate these claims and (importantly) related policy proposals

  • perceived fragility of banks is the justification for (costly) policy interventions

  • recent events are likely to spur new policies/regulations

  • need to understand the potential sources of instability

Q: What features of the environment allow self-fulfilling runs to occur?

  • some partial answers, but much remains unknown

  • we need a reliable “laboratory” to evaluate intuition and policy proposals
Literature following Diamond and Dybvig (1983):

- Jacklin (1987) and Wallace (1988) highlight the important of being explicit about the environment
  - agents are isolated; sequential service constraint

- Green & Lin (2003) study a model with sequential service
  - efficient allocation is uniquely implemented
  - self-fulfilling runs cannot occur under the optimal contract

- Peck and Shell (2003) do get runs in a similar environment

**Q:** What exactly is needed to generate a run equilibrium in a fully-specified model of financial intermediation?
What We Do

- Study a generalized version of the Green-Lin model
  - allow correlation in agents’ types
- Compute the efficient allocation for any number of agents
- Construct examples of run equilibria (surprising)
  - Green-Lin result is not robust to changes in distribution of types
- Clarify nature of the differences between results in Green-Lin and Peck-Shell
Environment

Green-Lin version of the Diamond-Dybvig model:

- 2 time periods, $t = 0, 1$
- Finite number $I$ of traders
- Traders are isolated from each other; markets cannot meet
  - can contact an intermediary in each period
- Intermediary has $I$ units of good in period 0
  - return on investment is $R > 1$ in period 1
Preferences

- Utility:
  \[
  v(a_i^0, a_i^1; \omega_i) = \frac{(a_i^0 + \omega_i a_i^1)^{1-\gamma}}{1 - \gamma} \quad \gamma > 1
  \]
  
  where \( \omega_i = \begin{cases} 0 \\ 1 \end{cases} \) if trader \( i \) is \{ impatient, patient \}

- Type \( \omega_i \) is private information

- \( \pi = \) probability of \( (\omega_i = 0) \)
  
  - types may be independent (Green & Lin) or correlated

- \( \omega = (\omega_1, \omega_2, \ldots, \omega_I) \) denotes the aggregate state of nature
Sequential Service

- At $t = 0$, traders contact the intermediary sequentially
  - idea used in Diamond-Dybvig, formalized by Wallace (1988)
  - order given by index $i$ (hence, known by traders)
- Traders must be paid as they arrive (an “urgent” need to consume)
- Sequential service constraint:
  \[
  a_i^0(\omega) = a_i^0(\hat{\omega}) \quad \text{for all } \omega, \hat{\omega} \text{ with } \omega^i = \hat{\omega}^i
  \]
  - $a_i^0$ can only depend on the information received by the intermediary prior to $i$
Allocations

- Set of feasible (ex post) allocations:
  \[ \mathcal{A} = \left\{ a : \mathbb{I} \rightarrow \mathbb{R}^2_+ \times \{0, 1\}^2 : \sum_{i \in \mathbb{I}} \left( a_i^0 + \frac{a_i^1}{R} \right) \leq I \right\} \]

- Set of feasible state-contingent allocations:
  \[ \mathcal{F} = \left\{ a : \{0, 1\}^I \rightarrow \mathcal{A} \right\} \]

- Efficient allocation \( a^* \) maximizes sum of expected utilities
  - subject to feasibility, sequential service

- Solving for the efficient allocation is a finite dynamic-programming problem
Efficient allocation

First: some obvious properties of the efficient allocation

(i) Impatient traders consume only at \( t = 0 \); patient traders only at \( t = 1 \)

\[
a_i^0 (\omega) = 0 \quad \text{if} \quad \omega_i = 1 \quad \text{and} \quad a_i^1 (\omega) = 0 \quad \text{if} \quad \omega_i = 0.
\]

(ii) Resources remaining at \( t = 1 \) are divided evenly among patient traders

\[
a_i^1 (\omega) = \frac{R \left( I - \sum_{i=1}^{I} c_i^0 (\omega) \right)}{\theta (\omega)}
\]

where

\[
\theta (\omega) = \sum_{i=1}^{I} \omega_i
\]
• All that remains is to determine $a_i^0(\omega)$ when $\omega_i = 0$
  • If trader $i$ is impatient, how much should she consume?

• Suppose intermediary has:
  • $y$ units of good left
  • encountered $\theta$ patient traders so far

• Let $V_i^\omega(y, \theta) =$ expected utility of traders $i, \ldots, I$
  • conditional on trader $i$ being type $\omega$

• These value functions must satisfy:
\[
V_i^0 (y_{i-1}, \theta_{i-1}) = \max \left\{ c_i^0 \right\} \begin{cases} 
\frac{(a_i^0)^{1-\gamma}}{1-\gamma} + p_{i+1} (\theta_{i-1}) V_{i+1}^0 (y_{i-1} - a_i^0, \theta_{i-1}) \\
+ (1 - p_{i+1} (\theta_{i-1})) V_{i+1}^1 (y_{i-1} - a_i^0, \theta_{i-1}) 
\end{cases}
\]

\[
V_i^1 (y_{i-1}, \theta_{i-1}) = \begin{cases} 
p_{i+1} (\theta_{i-1} + 1) V_{i+1}^0 (y_{i-1}, \theta_{i-1} + 1) + \\
(1 - p_{i+1} (\theta_{i-1} + 1)) V_{i+1}^1 (y_{i-1}, \theta_{i-1} + 1) 
\end{cases}
\]

- **Solution:**

\[
a_i^0 = \frac{y_{i-1}}{\psi_i (\theta_{i-1})^{1/\gamma} + 1}
\]

\[
\psi_i (x) = p_{i+1} (x) (\psi_{i+1} (x)^{1/\gamma} + 1)^\gamma + (1 - p_{i+1} (x)) \psi_{i+1} (x + 1)
\]

\[
\psi_i (x) = (x R^{1/\gamma})^\gamma
\]
• Example: $I = 4$, $R = 2$, $\gamma = 6$, $\pi = 0.5$ (independent)
Implementation

• Intermediary wants to implement the efficient allocation \(a^*\)
• Traders play a direct revelation game
  • contact intermediary sequentially and report type
  • receive payments according to efficient allocation
  • do not observe each others’ actions (isolation)
• Order in which traders contact intermediary is given by \(i\)
  • this order is known to traders (as in Green & Lin)
• Direct revelation game with strategies:

\[ \mu_i : \omega_i \mapsto \{0, 1\} \]

and payoffs:

\[ U_i (a^* \circ (\mu_{-i}, \mu_i)) \]

• Equilibrium: a profile \( \mu^* \) such that

\[ U_i (a^* \circ (\mu_{-i}^*, \mu_i^*)) \geq U_i (a^* \circ (\mu_{-i}^*, \mu_i)) \quad \forall \mu_i \forall i \]

• If \( a^* \) is incentive compatible, \( \mu^* = \omega \) is an equilibrium

  • Green & Lin show this always holds with independent types
The Question

Q: Does game have an equilibrium where $\mu_i^* \neq \omega_i$ for some $i$?

- any false reports must come from patient traders (i.e., a run)
- if so, a run can occur with positive probability in the “overall” game where intermediary chooses contract

Green & Lin’s result:

- When types are independent, answer is ‘no’
  - surprising; information frictions not “strong enough”
**Intuition for Green-Lin Result**

- Backward induction argument; start with trader $I$
  - regardless of reports of previous traders, she receives more consumption if she reports ‘patient’
  - reporting truthfully is a dominant strategy
- For any trader $i$: suppose everyone after her in line will report truthfully
  - G&L show she strictly prefers to report truthfully, regardless of reports before her (Lemma 5)
  - nontrivial property of the efficient allocation; “continuation IC”
- Iterated deletion of strictly dominated strategies leaves only truthful reporting for all $i$
Andolfatto, Nosal, & Wallace (2007)

- Suppose traders can observe earlier actions before reporting
  - change in environment; dynamic game
- Incentive compatibility in this environment is equivalent to “continuation IC” in Green-Lin
  - IC: trader $i$ is willing to report truthfully if all others do so, for any profile of $\omega^{i-1}$
  - any partial history of reports $\mu^{i-1}$ could have been truthful
  - trader $i$ is willing to report truthfully if everyone after him will do so, regardless of the actions of those before him (＝ continuation IC in Green-Lin)
- ANW’s main result: In this modified environment, any IC allocation can be uniquely implemented
  - same backward induction argument as before
  - also allow for more general preferences
- Like Green & Lin, this result relies on:
  - independent types (in fact, ANW highlight the importance of this assumption)
  - all traders report in period 0
- We work with the Green-Lin environment
Q: How important is backward induction to the G&L result?

- answer is not obvious

- Diamond-Dybvig and others generate runs using a **simple** contract
  - all early withdrawers receive same amount

- Is adding flexibility in the contract (as in G&L) enough to prevent runs?
  - or is the information depositors have about the order of withdrawals important?
• Peck & Shell (2003) address this issue
  
  • study a model with no restrictions on contracts other than sequential service

• Model differs from Green-Lin in two respects
  
  (i) agents must act before knowing position in order (an additional friction)

  (ii) preferences are different (marginal utility is type dependent)

• Construct examples of run equilibria
  
  • first examples in literature without ad hoc restrictions on contracts
Q: Is the difference in results due to

- the difference in information (backward induction)?
- the difference in preferences?

- We are able to answer this question
- Take the Green-Lin model with independent types
- Suppose traders must act before knowing \( i \)
  - expected utility

\[
\frac{1}{I} \sum_{i \in I} E[U_i(a, \omega)]
\]
• Model is exactly Green-Lin, but with Peck-Shell information structure

• Efficient allocation is unchanged
  • this is key: we can use our solution above

• We construct examples of run equilibria
  • easy when I is large

⇒ Peck-Shell results do not depend on their particular assumptions about preferences
One example: \( I = 15, \ R = 1.1, \ \gamma = 6, \ \pi = 0.1 \)

Figure: Expected utility if all other traders run
Correlated Types

- Return to Green-Lin model (traders know the order)

- Suppose $\omega_i$ are not i.i.d.
  - traders have private info about others’ types

- Example: $I = 4$, $R = 2$, $\gamma = 6$

  number of impatient traders

<table>
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<tr>
<th>number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
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<td>0.01</td>
<td>0.96</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

- Example is “close” to a model with no aggregate uncertainty
  - useful for gaining intuition; not important in general
• Efficient allocation:

• large payments for first two early withdrawals \( \sim c_E^* \)

• much lower payments if \( > 2 \) early withdrawals
A Run Equilibrium?

- Trader I will always report truthfully (as in Green & Lin)
  - any run equilibrium must be *partial*

- Result: the following strategies are an equilibrium:

\[ \mu_i^* = \begin{cases} 0 & \text{for } i = \{1 \text{ and } 2\} \\ \omega_i & \text{for } i = \{3 \text{ and } 4\} \end{cases} \]

- first two traders in the order run
- last two traders report truthfully

- Critical question: Why does trader 2 run?
  - why does the backward induction argument break down?
• Trader 2 knows trader 1 has withdrawn
  • will be 2nd withdrawal if she runs \( \sim c^*_E \)
  • if she waits, consumption depends on \( \omega_3 \) and \( \omega_4 \)
  • if \( \omega_3 = \omega_4 = 0 \), her consumption will be low \( \left( < c^*_E \right) \)

• Planner treats trader 1’s report as truthful
  • very unlikely that both 3 & 4 are impatient

• Trader 2 knows trader 1’s report was uninformative
  • very possible that both 3 & 4 are impatient
Given trader 2’s beliefs, the early payment \( (\sim c_E^*) \) is attractive

- the “continuation IC” property fails here

Reason: traders have better information about the types of the remaining agents

- and, thus, about additional early withdrawals

Information frictions keep this info from the intermediary

- result: intermediary is too optimistic, sets \( c_i^0 \) too high

Note: this cannot happen when types are independent

Easy to construct examples with more traders, etc.
Another example

- Suppose:

- significant aggregate uncertainty (but extreme values are unlikely)
The following strategies are an equilibrium

\[ \mu_i^* = \begin{cases} 0 & \text{for } i = \{1, \ldots, 7\} \\ \omega_i & \text{for } i = \{8, 9, 10\} \end{cases} \]

- first seven traders in the order run
- last three traders report truthfully

Trader 7 is the “critical” trader:
- in equilibrium, she thinks intermediary is overly-optimistic about likelihood withdrawals after her (same logic as before)
Summary

- In general, traders only run if they expect more early withdrawals than intermediary had planned for.

- Green & Lin: traders know positions in the order
  - all that matters is number of *additional* early withdrawals
  - the last trader will always report truthfully

- A run equilibrium requires a “critical” trader (last to run)
  - will run if she is more pessimistic than the intermediary about additional early withdrawals

- How can this arise in equilibrium?
• Number of additional early withdrawals depends on:
  • number of traders remaining in the order
  • probability distribution over their types

• A run requires that – in equilibrium – the critical trader is more pessimistic than the intermediary

• With independent types, this cannot occur
  • types of remaining agents are unrelated to those who have withdrawn

• We show that when types are correlated, it can occur
Extensions

- Suppose intermediary only observes *withdrawal requests*
  - traders who are not withdrawing stay at home

- Changes the efficient allocation
  - intermediary has less information to condition payments on
  - we compute using a similar dynamic programming problem

- We show: run equilibria exist even with independent types
  - again, critical trader is pessimistic about the number of additional early withdrawals
  - another dimension in which unique-implementation result is not robust
Concluding Remarks

- Green & Lin derived a remarkable result:
  - in a Diamond-Dybvig-style model, the efficient allocation is uniquely implementable
  - self-fulfilling runs are not possible

- The backward-induction logic seemed very general
  - tempting to draw the conclusion that self-fulfilling runs cannot occur if contract is designed optimally

- We show that introducing correlation in types overturns the unique-implementation result
  - the possibility of self-fulfilling runs cannot be ruled out on theoretical grounds
Extra Stuff
Commitment

- Consider the overall game, including the choice of contract
  - intermediary moves first, then traders play withdrawal game

- A run cannot occur with certainty in this game
  - if intermediary knows traders will run, would choose a “run proof” payment schedule
  - one possibility: $x_n = 1$ for all $n$

- However, a run could occur with some probability
  - traders coordinate on a “sunspot” variable; correlated eqm

- What is the maximum probability of a run consistent with equilibrium?
  - straightforward to show $> 0$; continuity argument
• Maximum probability of a run depends on the welfare difference between $x^*$ and the best “run proof” contract

• One possibility: “suspension of convertibility”

\[ x_n = \begin{cases} 
  x_n^* \\
  0 
\end{cases} \quad \text{for} \quad n \begin{cases} 
  \leq \\
  \geq 
\end{cases} (\pi + \epsilon) I \]

• Clearly generates lower welfare than $x^*$, but ...
  • welfare converges to that under $x^*$ as $I \to \infty$

• Conjecture: With $\delta_2 \gg 0$ and independent types, the maximum probability of a run $\to 0$ as $I \to \infty$
  • bank runs should not be a significant concern when $I$ is large
• However, this assumes the intermediary can commit to the payment schedule

• Ennis and Keister (2007): In an environment without commitment, runs can occur even when I is very large
  - suspending payments is *ex post* inefficient
  - lack of commitment leads intermediary to respond to a run with a partial suspension
  - broadly similar to the efficient payment schedule studied here

• Result relies on the costly communication friction
  - delays flow of information to the intermediary
  - intermediary is slow to recognize that a run is underway