Run Equilibria in the Green-Lin Model of Financial Intermediation

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Introduction

- Financial intermediaries are commonly believed to be inherently "fragile"
- Take short-term deposits, make long-term investments
- Result: illiquidity
  - short-term liabilities $>$ short-term assets
- If all investors withdraw funds at once, intermediary will fail
  - if intermediary will fail, investors want to withdraw

$\Rightarrow$ hints at possibility of a self-fulfilling bank run

- Classic model: Diamond & Dybvig (1983)

- Maturity transformation/illiquidity is not limited to banks
  - also performed by other financial institutions and in markets
- Examples:
  - Asset-backed commercial paper
  - Money-market/cash management funds
  - Auction-rate securities
  - Investment banks (Bear Stearns, Lehman Bros.)
- Many recent events appear "similar" to a bank run
  - Eichengreen: "What happened to Bear Stearns ... looked a lot like a 19th century run on the bank."

- Want to be able to evaluate these claims and (importantly) related policy proposals
  - perceived fragility of banks is the justification for (costly) policy interventions
  - recent events are likely to spur new policies/regulations
  - need to understand the potential sources of instability

Q: What features of the environment allow self-fulfilling runs to occur?

  - some partial answers, but much remains unknown
  - we need a reliable "laboratory" to evaluate intuition and policy proposals

Literature following Diamond and Dybvig (1983):

- Jacklin (1987) and Wallace (1988) highlight the important of being explicit about the environment
  - agents are isolated; sequential service constraint
- Green & Lin (2003) study a model with sequential service
  - efficient allocation is uniquely implemented
  - self-fulfilling runs cannot occur under the optimal contract
- Peck and Shell (2003) do get runs in a similar environment

Q: What exactly is needed to generate a run equilibrium in a fully-specified model of financial intermediation?

What We Do

- Study a generalized version of the Green-Lin model
  - allow correlation in agents' types
- Compute the efficient allocation for any number of agents
- Construct examples of run equilibria (surprising)

  $\Rightarrow$ Green-Lin result is not robust to changes in distribution of types
- Clarify nature of the differences between results in Green-Lin and Peck-Shell

Environment

Green-Lin version of the Diamond-Dybvig model:

- 2 time periods, $t = 0, 1$
- Finite number I of traders
- Traders are isolated from each other; markets cannot meet
    - can contact an intermediary in each period
- Intermediary has I units of good in period 0
    - return on investment is $R > 1$ in period 1

Preferences

- Utility:

$$v\left(a_i^0, a_i^1; \omega_i\right) = \frac{\left(a_i^0 + \omega_i a_i^1\right)^{1-\gamma}}{1-\gamma} \qquad \gamma > 1$$

$$\text{where } \omega_i = \left\{ \begin{array}{c} 0 \\ 1 \end{array} \right\} \text{ if trader } i \text{ is } \left\{ \begin{array}{l} \text{impatient} \\ \text{patient} \end{array} \right\}$$

- Type ω_i is private information
- π = probability of $(\omega_i = 0)$
    - types may be independent (Green & Lin) or correlated
- $\omega = (\omega_1, \omega_2, \ldots, \omega_I)$ denotes the aggregate state of nature

Sequential Service

- At $t = 0$, traders contact the intermediary sequentially
  - idea used in Diamond-Dybvig, formalized by Wallace (1988)
  - order given by index i (hence, known by traders)
- Traders must be paid as they arrive (an "urgent" need to consume)
- Sequential service constraint:

$$a_i^0(\omega) = a_i^0(\widehat{\omega}) \text{ for all } \omega, \widehat{\omega} \text{ with } \omega^i = \widehat{\omega}^i$$

  - a_i^0 can only depend on the information received by the intermediary prior to i

Allocations

- Set of feasible (ex post) allocations:

$$\mathbb{A} = \left\{ \ a : \mathbb{I} \rightarrow \mathbb{R}_+^2 \times \{0,1\}^2 : \textstyle\sum_{i \in \mathbb{I}} \left(a_i^0 + \frac{a_i^1}{R} \right) \leq \mathrm{I} \quad \right\}$$

- Set of feasible state-contingent allocations:

$$\mathbb{F} = \left\{ \mathbf{a} : \{0,1\}^{\mathrm{I}} \rightarrow \mathbb{A} \right\}$$

- Efficient allocation $\mathbf{a}^*$ maximizes sum of expected utilities
  - subject to feasibility, sequential service
- Solving for the efficient allocation is a finite dynamic-programming problem

Efficient allocation

First: some obvious properties of the efficient allocation

(*i*) Impatient traders consume only at $t = 0$; patient traders only at $t = 1$

$$\mathbf{a}_i^0(\omega) = 0 \text{ if } \omega_i = 1 \quad \text{and} \quad \mathbf{a}_i^1(\omega) = 0 \text{ if } \omega_i = 0.$$

(*ii*) Resources remaining at $t = 1$ are divided evenly among patient traders

$$\mathbf{a}_i^1(\omega) = \frac{R\left(\mathrm{I} - \sum_{i=1}^{\mathrm{I}} c_i^0(\omega)\right)}{\theta(\omega)}$$

where

$$\theta(\omega) = \sum_{i=1}^{\mathrm{I}} \omega_i$$

- All that remains is to determine $\mathbf{a}_i^0(\omega)$ when $\omega_i = 0$
  - If trader i is impatient, how much should she consume?
- Suppose intermediary has:
  - y units of good left
  - encountered θ patient traders so far
- Let $V_i^{\omega}(y, \theta) =$ expected utility of traders $i, \ldots, \mathrm{I}$
  - conditional on trader i being type ω
- These value functions must satisfy:

$$V_i^0\left(y_{i-1},\theta_{i-1}\right)=\max_{\left\{c_i^0\right\}}\left\{\begin{array}{c}\frac{\left(a_i^0\right)^{1-\gamma}}{1-\gamma}+p_{i+1}\left(\theta_{i-1}\right)V_{i+1}^0\left(y_{i-1}-a_i^0,\theta_{i-1}\right)\\ +\left(1-p_{i+1}\left(\theta_{i-1}\right)\right)V_{i+1}^1\left(y_{i-1}-a_i^0,\theta_{i-1}\right)\end{array}\right\}$$

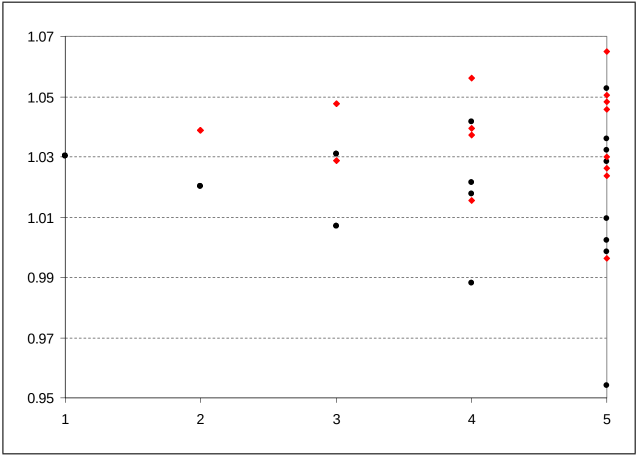
$$V_i^1\left(y_{i-1},\theta_{i-1}\right)=\left\{\begin{array}{c}p_{i+1}\left(\theta_{i-1}+1\right)V_{i+1}^0\left(y_{i-1},\theta_{i-1}+1\right)+\\ \left(1-p_{i+1}\left(\theta_{i-1}+1\right)\right)V_{i+1}^1\left(y_{i-1},\theta_{i-1}+1\right)\end{array}\right\}$$

- Solution:

$$a_i^0=\frac{y_{i-1}}{\psi_i\left(\theta_{i-1}\right)^{\frac{1}{\gamma}}+1}$$

$$\begin{aligned}\psi_i\left(x\right)&=p_{i+1}\left(x\right)\left(\psi_{i+1}\left(x\right)^{\frac{1}{\gamma}}+1\right)^{\gamma}+\left(1-p_{i+1}\left(x\right)\right)\psi_{i+1}\left(x+1\right)\\ \psi_I\left(x\right)&=\left(xR^{\frac{1-\gamma}{\gamma}}\right)^{\gamma}\end{aligned}$$

- Example: $\mathrm{I} = 4$, $R = 2$, $\gamma = 6$, $\pi = 0.5$ (independent)



Implementation

- Intermediary wants to implement the efficient allocation $\mathbf{a}^*$
- Traders play a direct revelation game
  - contact intermediary sequentially and report type
  - receive payments according to efficient allocation
  - do not observe each others' actions (isolation)
- Order in which traders contact intermediary is given by i
  - this order is known to traders (as in Green & Lin)

- Direct revelation game with strategies:

$$\mu_i : \omega_i \mapsto \{0, 1\}$$

  and payoffs:

$$U_i \left(\mathbf{a}^* \circ \left(\mu_{-i}, \mu_i \right) \right)$$

- Equilibrium: a profile μ^* such that

$$U_i \left(\mathbf{a}^* \circ \left(\mu^*_{-i}, \mu^*_i \right) \right) \geq U_i \left(\mathbf{a}^* \circ \left(\mu^*_{-i}, \mu_i \right) \right) \quad \forall \ \mu_i \ \forall i$$

- If $\mathbf{a}^*$ is incentive compatible, $\mu^* = \omega$ is an equilibrium
  - Green & Lin show this always holds with independent types

The Question

Q: Does game have an equilibrium where $\mu_i^* \neq \omega_i$ for some i?

- any false reports must come from patient traders (i.e., a run)
- if so, a run can occur with positive probability in the "overall" game where intermediary chooses contract

Green & Lin's result:

- When types are independent, answer is 'no'
  - surprising; information frictions not "strong enough"

Intuition for Green-Lin Result

- Backward induction argument; start with trader I
  - regardless of reports of previous traders, she receives more consumption if she reports 'patient'
  - reporting truthfully is a dominant strategy
- For any trader i : suppose everyone after her in line will report truthfully
  - G&L show she strictly prefers to report truthfully, regardless of reports before her (Lemma 5)
  - nontrivial property of the efficient allocation; "continuation IC"
- Iterated deletion of strictly dominated strategies leaves only truthful reporting for all i

Andolfatto, Nosal, & Wallace (2007)

- Suppose traders can observe earlier actions before reporting
  - change in environment; dynamic game
- Incentive compatibility in this environment is equivalent to "continuation IC" in Green-Lin
  - IC: trader i is willing to report truthfully if all others do so, for any profile of ω^{i-1}
  - any partial history of reports μ^{i-1} could have been truthful
  - trader i is willing to report truthfully if everyone after him will do so, regardless of the actions of those before him ($=$ continuation IC in Green-Lin)

- ANW's main result: In this modified environment, any IC allocation can be uniquely implemented
    - same backward induction argument as before
    - also allow for more general preferences
- Like Green & Lin, this result relies on:
    - independent types (in fact, ANW highlight the importance of this assumption)
    - all traders report in period 0
- We work with the Green-Lin environment

Early Decisions

Q: How important is backward induction to the G&L result?

- answer is not obvious

- Diamond-Dybvig and others generate runs using a **simple** contract
  - all early withdrawers receive same amount
- Is adding flexibility in the contract (as in G&L) enough to prevent runs?
  - or is the information depositors have about the order of withdrawals important?

- Peck & Shell (2003) address this issue
  - study a model with no restrictions on contracts other than sequential service
- Model differs from Green-Lin in two respects
  - (*i*) agents must act before knowing position in order (an additional friction)
  - (*ii*) preferences are different (marginal utility is type dependent)
- Construct examples of run equilibria
  - first examples in literature without ad hoc restrictions on contracts

Q: Is the difference in results due to

- the difference in information (backward induction)?
- the difference in preferences?

- We are able to answer this question
- Take the Green-Lin model with independent types
- Suppose traders must act before knowing i
  - expected utility

$$\frac{1}{\mathrm{I}} \sum_{i \in \mathbb{I}} E\left[U_i\left(\mathbf{a}, \omega\right)\right]$$

- Model is exactly Green-Lin, but with Peck-Shell information structure
- Efficient allocation is unchanged
  - this is key: we can use our solution above
- We construct examples of run equilibria
  - easy when I is large

$\Rightarrow$ Peck-Shell results do **not** depend on their particular assumptions about preferences

- One example: $\mathrm{I} = 15$, $R = 1.1$, $\gamma = 6$, $\pi = 0.1$

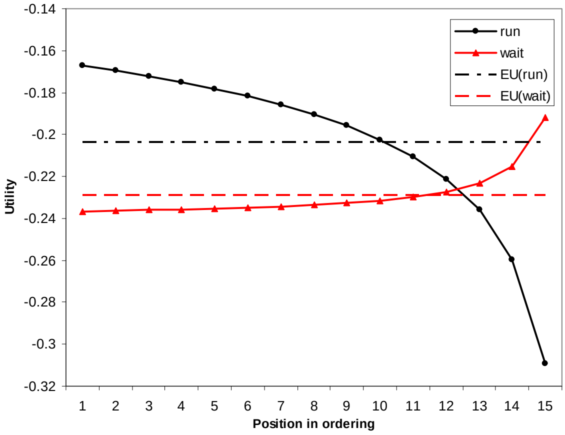


Figure: Expected utility if all other traders run

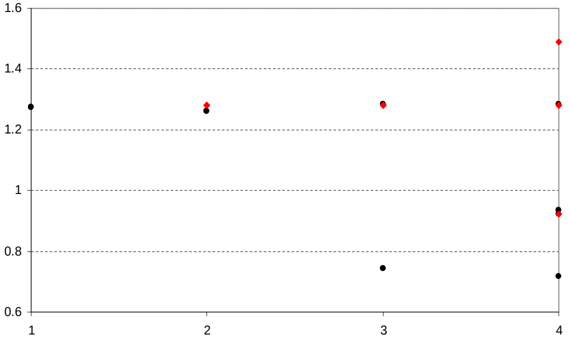
Correlated Types

- Return to Green-Lin model (traders know the order)
- Suppose ω_i are not i.i.d.
  - traders have private info about *others'* types
- Example: $\mathrm{I} = 4$, $R = 2$, $\gamma = 6$

	number of impatient traders				
	0	1	2	3	4
probability	0.01	0.01	0.96	0.01	0.01

- Example is "close" to a model with no aggregate uncertainty
  - useful for gaining intuition; not important in general

- Efficient allocation:



- large payments for first two early withdrawals $\left(\sim c_E^*\right)$
- much lower payments if > 2 early withdrawals

A Run Equilibrium?

- Trader I will always report truthfully (as in Green & Lin)
  - any run equilibrium must be *partial*
- Result: the following strategies are an equilibrium:

$$\mu_i^* = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } i = \left\{ \begin{array}{c} 1 \text{ and } 2 \\ 3 \text{ and } 4 \end{array} \right\}$$

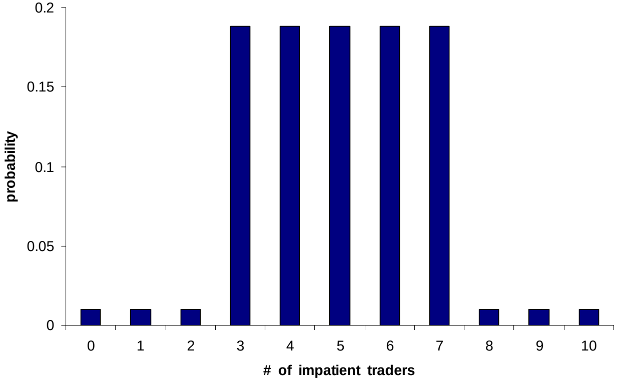
  - first two traders in the order run
  - last two traders report truthfully
- Critical question: Why does trader 2 run?
  - why does the backward induction argument break down?

- Trader 2 knows trader 1 has withdrawn
  - will be 2nd withdrawal if she runs $(\sim c_E^*)$
  - if she waits, consumption depends on ω_3 and ω_4
  - if $\omega_3 = \omega_4 = 0$, her consumption will be low $(< c_E^*)$
- Planner treats trader 1's report as truthful
  - very unlikely that both 3 & 4 are impatient
- Trader 2 knows trader 1's report was uninformative
  - very possible that both 3 & 4 are impatient

- Given trader 2's beliefs, the early payment $(\sim c_E^*)$ is attractive
  - the "continuation IC" property fails here
- Reason: traders have better information about the types of the remaining agents
  - and, thus, about additional early withdrawals
- Information frictions keep this info from the intermediary
  - result: intermediary is too optimistic, sets c_i^0 too high
- Note: this cannot happen when types are independent
- Easy to construct examples with more traders, etc.

Another example

- Suppose:



  - significant aggregate uncertainty (but extreme values are unlikely)

- The following strategies are an equilibrium

$$\mu_i^* = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } i = \left\{ \begin{array}{c} 1, ..., 7 \\ 8, 9, 10 \end{array} \right\}$$

  - first seven traders in the order run
  - last three traders report truthfully

- Trader 7 is the "critical" trader:
  - in equilibrium, she thinks intermediary is overly-optimistic about likelihood withdrawals after her (same logic as before)

Summary

- In general, traders only run if they expect more early withdrawals than intermediary had planned for
- Green & Lin: traders know positions in the order
    - all that maters it number of *additional* early withdrawals
    - the last trader will always report truthfully
- A run equilibrium requires a "critical" trader (last to run)
    - will run if she is more pessimistic than the intermediary about additional early withdrawals
- How can this arise in equilibrium?

- Number of additional early withdrawals depends on:
  - number of traders remaining in the order
  - probability distribution over their types
- A run requires that – in equilibrium – the critical trader is more pessimistic than the intermediary
- With independent types, this cannot occur
  - types of remaining agents are unrelated to those who have withdrawn
- We show that when types are correlated, it can occur

Extensions

- Suppose intermediary only observes *withdrawal requests*
  - traders who are not withdrawing stay at home
- Changes the efficient allocation
  - intermediary has less information to condition payments on
  - we compute using a similar dynamic programming problem
- We show: run equilibria exist even with independent types
  - again, critical trader is pessimistic about the number of additional early withdrawals
  - another dimension in which unique-implementation result is not robust

Concluding Remarks

- Green & Lin derived a remarkable result:
    - in a Diamond-Dybvig-style model, the efficient allocation is uniquely implementable
    - self-fulfilling runs are not possible
- The backward-induction logic seemed very general
    - tempting to draw the conclusion that self-fulfilling runs cannot occur if contract is designed optimally
- We show that introducing correlation in types overturns the unique-implementation result
    - the possibility of self-fulfilling runs cannot be ruled out on theoretical grounds

Extra Stuff

Commitment

- Consider the overall game, including the choice of contract
  - intermediary moves first, then traders play withdrawal game
- A run cannot occur with certainty in this game
  - if intermediary knows traders will run, would choose a "run proof" payment schedule
  - one possibility: $x_n = 1$ for all n
- However, a run could occur with some probability
  - traders coordinate on a "sunspot" variable; correlated eqm
- What is the maximum probability of a run consistent with equilibrium?
  - straightforward to show > 0; continuity argument

- Maximum probability of a run depends on the welfare difference between x^* and the best "run proof" contract

- One possibility: "suspension of convertibility"

$$x_n = \left\{ \begin{array}{c} x_n^* \\ 0 \end{array} \right\} \text{ for } n \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} (\pi + \varepsilon)\,\mathrm{I}$$

- Clearly generates lower welfare than x^*, but ...
  - welfare converges to that under x^* as $\mathrm{I} \to \infty$

- Conjecture: With $\delta_2 >> 0$ and independent types, the maximum probability of a run $\to 0$ as $\mathrm{I} \to \infty$
  - bank runs should not be a significant concern when I is large

- However, this assumes the intermediary can commit to the payment schedule
- Ennis and Keister (2007): In an environment without commitment, runs can occur even when I is very large
    - suspending payments is *ex post* inefficient
    - lack of commitment leads intermediary to respond to a run with a partial suspension
    - broadly similar to the efficient payment schedule studied here
- Result relies on the costly communication friction
    - delays flow of information to the intermediary
    - intermediary is slow to recognize that a run is underway