Supplemental Appendix

The following is a supplemental appendix to Ennis and Keister (2006) that explains in detail where the problems in the analysis of Cooper and Ross (1998) occur and how our approach addresses them. There are two problems related to their Proposition 3 (p.36). The first one is easily fixed, but helps explain where the second, more fundamental, problem arises. We use the same notation as in the Cooper-Ross paper, and the two numbered equations below (CR-5 and CR-6) are identical to the correspondingly-numbered equations in that paper.

S.1 The Bank's Problem(s)

The paper does not write down a single optimization problem for the bank. Rather, it considers two problems and compares the solutions of these two problems to find the optimal contract for the bank to offer. The first problem defines the *best run-proof contract* (see p.34):

$$\begin{aligned}
&\max\ \pi U(c_E) + (1-\pi)U(c_L) \\
&\text{s.t.} \\
&\qquad \pi c_E = 1 - i - i_2 \\
&\qquad (1-\pi)c_L = iR + i_2 \\
&\qquad c_E \leq 1 - i\tau.
\end{aligned} \tag{CR-5}$$

The second problem defines the *best runs-permitting contract* (see p. 35):

$$\begin{aligned}
&\max\ (1-q)\left[\pi U(c_E) + (1-\pi)U(c_L)\right] + qU(c_E)\frac{1-i\tau}{c_E} \\
&\text{s.t.} \\
&\qquad \pi c_E = 1 - i - i_2 \\
&\qquad (1-\pi)c_L = iR + i_2 \\
&\qquad i_2 \geq 0
\end{aligned} \tag{CR-6}$$

Whichever of these contracts yields a higher value of its respective objective function is then the optimal banking contract. The issue is what this optimal contract looks like when the following condition on parameters holds:

$$q\tau > (1-q)(R-1) \tag{Q}$$

S.2 The Objective Function in (6)

The first problem with the analysis is in the statement of problem (6). To see this, consider solving the problem as stated. The working paper version (Cooper and Ross, 1991, p.18) lists the first-order conditions for this problem as:

$$
\begin{aligned}
(1-q)\,\pi U'(c_E) + q\frac{1-i\tau}{c_E}\left(U'(c_E) - \frac{U(c_E)}{c_E}\right) &= \lambda^E \pi && \text{(a)}\\
(1-q)(1-\pi)\,U'(c_L) &= \lambda^L (1-\pi) && \text{(b)}\\
\frac{q\tau}{c_E}U(c_e) + \lambda^E &= \lambda^L R && \text{(c)}\\
\lambda^L + \beta &= \lambda^E && \text{(d)}
\end{aligned}
$$

where λ^E, λ^L, and β are the multipliers on the constraints in the order they are listed above. We begin by showing that when (Q) holds, the best runs-permitting contract is *not* characterized by these first-order conditions.

Remark 1: Under condition (Q), the above first-order conditions do not have a solution.

Proof: Combining (a) and (b) yields

$$\lambda^E - \lambda^L = (1-q)\left[U'(c_E) - U'(c_L)\right] + q\frac{1-i\tau}{\pi c_E}\left[U'(c_E) - \frac{U(c_E)}{c_E}\right].$$

Since $\beta \geq 0$ must hold, (d) implies $\lambda^E \geq \lambda^L$ and hence that the expression on the right above must be non-negative. Concavity of U and the normalization $U(0) = 0$ (which was used in the statement of problem (6)) imply that the final term (in square brackets) is negative; we must therefore have

$$U'(c_E) - U'(c_L) > 0.$$

Hence any solution to the first-order conditions must satisfy $c_E < c_L$. However, note that (c) and (d) together imply

$$q\tau\frac{U(c_e)}{c_E} + \beta = \lambda^L (R-1)$$

Substituting for λ^L using (b) then yields

$$q\tau\frac{U(c_E)}{c_E} + \beta = (1-q)(R-1)\,U'(c_L)\,. \tag{e}$$

Since we know that $c_E < c_L$ holds and that U is strictly concave with $U(0) = 0$, we must have

$$\frac{U(c_E)}{c_E} > U'(c_L).$$

Using this inequality and condition (Q) in equation (e) shows that β must be negative, a contradiction. ■

In fact, it is easy to see by looking at the objective function that the problem as written does not have a solution. Since $U(0) = 0$ and $U'(0) = \infty$ are assumed to hold, letting c_E go to zero makes the last term in the objective go to infinity. The last element of this term, $(1 - i\tau)/c_E$, is supposed to represent the probability of a typical agent being served during a run, and hence should not be larger than one. In other words, the objective function should be written as[1]

$$(1-q)\left[\pi U(c_E) + (1-\pi) U(c_L)\right] + qU(c_E) \min\left(\frac{1-i\tau}{c_E}, 1\right).$$

With this new objective function, if we consider the region where

$$\frac{1-i\tau}{c_E} < 1$$

holds, the first-order conditions are the same as above. Our argument above therefore shows that there is no solution in this region, and hence (under condition (Q)), the solution to the corrected problem must have

$$\frac{1-i\tau}{c_E} = 1.$$

That is, under condition (Q), the contract that solves (the corrected) problem (6) also lies in the constraint set of problem (5).

S.3 Runs-Permitting vs. Run-Proof

Proposition 3 in the published paper (p.36) states that, under condition (Q), the solution to (6) has $i_2 > 0$.[2] Once the *min* is placed in the objective function, this statement is true. However, the second (and more fundamental) problem with the analysis is that this statement is irrelevant for characterizing the optimal banking contract. As we have seen, when (Q) holds the solution to (6) also lies in the constraint set of (5). In other words, when the probability of a run is very high, the

[1] In our problem (P), the indicator function in the objective both (i) takes the place of the *min* used here, and (ii) allows us to write a single optimization problem that combines the Cooper-Ross problems (5) and (6).

[2] In the working paper version (1991), this result is Proposition 5 on p. 20.

bank wants to hold a portfolio that is so liquid that it actually prevents runs. Given that it is doing so, the bank should recognize that the relevant objective function is that in (5), *not* that in (6), and hence the optimal banking contract is the best run-proof contract.

Remark 2: Under condition (Q), the optimal banking contract is the solution to (5).

This is the content of our Proposition 2. Hence the Cooper-Ross analysis does *not* give conditions under which the solution to the bank's complete optimization problem involves $i_2 > 0$. We go on to ask under what conditions will a bank choose a contract that is runs-permitting and has $i_2 > 0$. Under the functional form assumption $u(c) = \frac{1}{\alpha}c^{\alpha}$, we show that this *never* happens (our Proposition 3). The reason that the bank never chooses an "interior" value of i_2 is that the objective function in (6) is not concave. When (Q) does not hold, the first-order conditions listed above can have a solution in the relevant region, but this solution always represents a local *minimum*.

S.4 Conclusions

Although the statement of Proposition 3 in the published paper is technically correct (once the *min* is added to the objective function), the fact that the paper did not realize Remark 2 above leads it to draw incorrect conclusions. For example, on page 37 the last sentence before Section 4 says: "If a contract allowing runs is chosen, then for τ large enough, excess liquidity will be held." The idea behind this statement (as we understand it) is the following. First, by choosing q small enough (below a cutoff level q^*), one can be sure that the optimal contract permits runs. Then by choosing τ large enough for (Q) to hold, one can be sure that the best runs-permitting contract has $i_2 > 0$. The problem with this argument is that the cutoff level q^* depends on τ. In fact, Remark 2 shows that whenever q and τ are such that condition (Q) holds, q is necessarily above the cutoff q^*.

References

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