Supplemental Appendix

The following is a supplemental appendix to “Bank Runs and Institutions: The Perils of Intervention” by Huberto M. Ennis and Todd Keister. This appendix contains the derivations behind two statements made in the main text.

**Proof of Lemma 1 (p.13):** Define the function $c_L(\pi_s)$ as in (5). This function gives the payoff to a patient depositor who waits until period 2 to withdraw when (i) all other patient depositors attempt to withdraw in period 1 and (ii) the BA declares a deposit freeze after a proportion $\pi_s$ of depositors have withdrawn. Note that we have

$$c_L(\pi) = \frac{c_L^*}{(1 - \pi)} > c_L^* > c_E^*.$$  

It is straightforward to show that $dc_L(\pi_s)/d\pi_s < 0$ holds. In addition, we know that $c_L(\pi_s^U) = 0$ for $\pi_s^U < 1$. Hence, there is a unique value $\pi_T$ such that $\pi_s < \pi_T$ implies $c_L(\pi_s) > c_E^*$, while $\pi_s > \pi_T$ implies $c_L(\pi_s) < c_E^*$. (See Figure 2.) Therefore, waiting is a strictly dominant strategy for patient depositors if and only if $\pi_s \in [\pi, \pi_T)$.

**Derivations for alternate utility function (p.14):** The text claims: “It can be shown that for any $\gamma > 1$, the condition for fragility (6) is satisfied under this utility function as long as $b$ is small enough.” This appendix provides the calculations behind this claim. Consider the utility function

$$u(c) = \frac{(c + b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}$$

with $b > 0$. Note that $u(0) = 0$. The coefficient of relative risk aversion is given by:

$$\rho(c) = -\frac{u''(c)c}{u'(c)} = \gamma \frac{c}{c + b} < \gamma$$

for all $b > 0$.

Hence, the coefficient of relative risk aversion is not constant. For a given value of $c$, however, we have that $\rho(c) \rightarrow \gamma$ as $b \rightarrow 0$.

With this alternative utility function, the solution to problem (1) is given by

$$i^* = \frac{(1 - \pi) R^{1-\gamma}}{\pi (1 - \pi) \left( R^{1-\gamma} \right)} + \frac{\pi (1 - \pi) \left( R^{1-\gamma} - 1 \right) b}{\pi R + (1 - \pi) R^{1-\gamma}}.$$
\[ c^*_E = \frac{1}{\pi + (1 - \pi) R^{1 - \gamma}} - \frac{(1 - \pi) \left( R^{1 - \gamma} - 1 \right)}{\pi R + (1 - \pi) R^{1 - \gamma}} b, \]

\[ c^*_L = \frac{R^{1 - \gamma}}{\pi + (1 - \pi) R^{1 - \gamma}} + \frac{R \pi \left( R^{1 - \gamma} - 1 \right)}{\pi R + (1 - \pi) R^{1 - \gamma}} b. \]

Assumption 1, the *illiquidity condition* (that is, \( c^*_E > 1 - \tau_0 \)), is now equivalent to the following condition:

\[ b < \frac{R \left[ 1 - (1 - \tau) \left( R^{1 - \gamma} \right) \right]}{(1 - \pi \tau) \left( R^{1 - \gamma} - 1 \right)}. \] (IC)

Note that since \( b > 0 \) this condition can only hold if \((1 - \tau) R^{1 - \gamma} > 1\), as we required in the main text. In fact, if \((1 - \tau) R^{1 - \gamma} > 1\) holds, then there always exists a small enough value of \( b \) such that the illiquidity condition (IC) holds.

Proposition 2 requires

\[ \pi - \frac{u' \left( c^*_E \right) c^*_E}{u \left( c^*_E \right)} \left[ \frac{R}{1 - \tau} - (1 - \pi) \right] \geq 0, \]

which can be written as

\[ \frac{u' \left( c^*_E \right) c^*_E}{u \left( c^*_E \right)} \leq \frac{\pi}{R^{1 - \gamma} - (1 - \pi)}. \]

Note that, in general, we have

\[ \frac{u' \left( c \right) c}{u \left( c \right)} = (1 - \gamma) \frac{(c + b)^{-\gamma} c}{(c + b)^{1 - \gamma} - b^{1 - \gamma}}. \]

If \( \gamma \in (0, 1) \) then we have

\[ \lim_{b \to 0} \frac{u' \left( c \right) c}{u \left( c \right)} = (1 - \gamma). \]

For this case, condition (6) in the main text immediately follows, after some simple rearrangements.

For the case of \( \gamma > 1 \), suppose we rewrite \( u' \left( c \right) c/u \left( c \right) \) as

\[ \frac{u' \left( c \right) c}{u \left( c \right)} = (\gamma - 1) \frac{1}{b^{\gamma - 1} - \left( c + b \right)^{\gamma - 1}}. \]
Since $c^*_E$ converges to a positive number as $b \to 0$ it is easy to see that

$$\lim_{b \to 0} \frac{u'(c^*_E) c^*_E}{u(c^*_E)} = 0.$$ 

Hence, for any $\gamma > 1$ (given all other parameter values) there exists a threshold value $\bar{b}(\gamma)$ such that $b < \bar{b}(\gamma)$ implies

$$\frac{u'(c^*_E) c^*_E}{u(c^*_E)} \leq \frac{\pi}{\frac{R}{1-\tau} - (1 - \pi)},$$

and, thus, (6) holds. \qed