Supplemental Appendix

The following is a supplemental appendix to "Bank Runs and Institutions: The Perils of Intervention" by Huberto M. Ennis and Todd Keister. This appendix contains the derivations behind two statements made in the main text.

Proof of Lemma 1 (p.13): Define the function $c_L(\pi_s)$ as in (5). This function gives the payoff to a patient depositor who waits until period 2 to withdraw when (i) all other patient depositors attempt to withdraw in period 1 and (ii) the BA declares a deposit freeze after a proportion π_s of depositors have withdrawn. Note that we have

$$c_L(\pi) = \frac{c_L^*}{(1-\pi)} > c_L^* > c_E^*.$$

It is straightforward to show that $dc_L(\pi_s)/d\pi_s < 0$ holds. In addition, we know that $c_L(\pi_s^U) = 0$ for $\pi_s^U < 1$. Hence, there is a unique value π^T such that $\pi_s < \pi^T$ implies $c_L(\pi_s) > c_E^*$, while $\pi_s > \pi^T$ implies $c_L(\pi_s) < c_E^*$. (See Figure 2.) Therefore, waiting is a strictly dominant strategy for patient depositors if and only if $\pi_s \in [\pi, \pi^T)$. $\square$

Derivations for alternate utility function (p.14): The text claims: "It can be shown that for any $\gamma > 1$, the condition for fragility (6) is satisfied under this utility function as long as b is small enough." This appendix provides the calculations behind this claim. Consider the utility function

$$u(c) = \frac{(c+b)^{1-\gamma} - b^{1-\gamma}}{1-\gamma}$$

with $b > 0$. Note that $u(0) = 0$. The coefficient of relative risk aversion is given by:

$$\rho(c) = -\frac{u''(c)\,c}{u'(c)} = \gamma\frac{c}{c+b} < \gamma \text{ for all } b > 0.$$

Hence, the coefficient of relative risk aversion is not constant. For a given value of c, however, we have that $\rho(c) \to \gamma$ as $b \to 0$.

With this alternative utility function, the solution to problem (1) is given by

$$i^* = \frac{(1-\pi)\,R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi)\,R^{\frac{1-\gamma}{\gamma}}} + \frac{\pi(1-\pi)\left(R^{\frac{1}{\gamma}} - 1\right)}{\pi R + (1-\pi)\,R^{\frac{1}{\gamma}}}b,$$

$$c_E^* = \frac{1}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} - \frac{(1-\pi)\left(R^{\frac{1}{\gamma}} - 1\right)}{\pi R + (1-\pi) R^{\frac{1}{\gamma}}} b,$$

$$c_L^* = \frac{R^{\frac{1}{\gamma}}}{\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}} + \frac{R\pi\left(R^{\frac{1}{\gamma}} - 1\right)}{\pi R + (1-\pi) R^{\frac{1}{\gamma}}} b.$$

Assumption 1, the *illiquidity condition* (that is, $c_E^* > 1 - \tau i^*$), is now equivalent to the following condition:

$$b < \frac{R\left[1 - (1-\tau) R^{\frac{1-\gamma}{\gamma}}\right]}{(1-\pi\tau)\left(R^{\frac{1}{\gamma}} - 1\right)}. \tag{IC}$$

Note that since $b > 0$ this condition can only hold if $(1-\tau) R^{\frac{1-\gamma}{\gamma}} > 1$, as we required in the main text. In fact, if $(1-\tau) R^{\frac{1-\gamma}{\gamma}} > 1$ holds, then there always exists a small enough value of b such that the illiquidity condition (IC) holds.

Proposition 2 requires

$$\pi - \frac{u'(c_E^*)\, c_E^*}{u(c_E^*)}\left[\frac{R}{1-\tau} - (1-\pi)\right] \geq 0,$$

which can be written as

$$\frac{u'(c_E^*)\, c_E^*}{u(c_E^*)} \leq \frac{\pi}{\frac{R}{1-\tau} - (1-\pi)}.$$

Note that, in general, we have

$$\frac{u'(c)\, c}{u(c)} = (1-\gamma)\frac{(c+b)^{-\gamma} c}{(c+b)^{1-\gamma} - b^{1-\gamma}}.$$

If $\gamma \in (0,1)$ then we have

$$\lim_{b\to 0}\frac{u'(c)\, c}{u(c)} = (1-\gamma).$$

For this case, condition (6) in the main text immediately follows, after some simple rearrangements.

For the case of $\gamma > 1$, suppose we rewrite $u'(c)\, c/u(c)$ as

$$\frac{u'(c)\, c}{u(c)} = (\gamma - 1)\frac{\frac{1}{(c+b)^{\gamma}} c}{\frac{1}{b^{\gamma-1}} - \frac{1}{(c+b)^{\gamma-1}}}.$$

Since c_E^* converges to a positive number as $b \to 0$ it is easy to see that

$$\lim_{b \to 0} \frac{u'\left(c_E^*\right) c_E^*}{u\left(c_E^*\right)} = 0.$$

Hence, for any $\gamma > 1$ (given all other parameter values) there exists a threshold value $\overline{b}\left(\gamma\right)$ such that $b < \overline{b}\left(\gamma\right)$ implies

$$\frac{u'\left(c_E^*\right) c_E^*}{u\left(c_E^*\right)} \leq \frac{\pi}{\frac{R}{1-\tau} - \left(1 - \pi\right)},$$

and, thus, (6) holds. $\square$