

# Bailouts and Financial Fragility

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## Abstract

Should policy makers be prevented from bailing out investors in the event of a crisis? I study this question in a model of financial intermediation with limited commitment. When a crisis occurs, the policy maker will respond with fiscal transfers that partially cover intermediaries' losses. The anticipation of this bailout distorts ex ante incentives, leading intermediaries to become excessively illiquid and increasing financial fragility. Prohibiting bailouts is not necessarily desirable, however: while it induces intermediaries to become more liquid, it may nevertheless lower welfare and leave the economy more susceptible to a crisis. A policy of taxing short-term liabilities, in contrast, can both improve the allocation of resources and promote financial stability.

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# 1 Introduction

The recent financial crisis has generated a heated debate about the effects of public-sector bailouts of distressed financial institutions. Most observers agree that the anticipation of being bailed out in the event of a crisis distorts the incentives faced by these institutions and their investors. By insulating these agents from the full consequences of a negative outcome, an anticipated bailout leads to a misallocation of resources and encourages risky behavior that may leave the economy more susceptible to a crisis. Opinions differ widely, however, on the best way to deal with this problem.

Some observers argue that the focus should be on restricting future policy makers' ability to engage in bailouts. Such a restriction would encourage financial institutions to provision for bad outcomes and, it is claimed, these actions would collectively make the financial system more stable. Swagel (2010), for example, argues "[a] resolution regime that provides certainty against bailouts will reduce the riskiness of markets and thus help avoid a future crisis." Similarly, Dijsselbloem (2013) states that "[b]y ensuring that the private sector bears the primary responsibility for bank resolution, market discipline will be increased and a sustainable, healthy financial sector can be achieved." Rehm (2013) reports that one Federal Reserve Bank president "wants to tie the government's hands so that regulators have no funds to finance these bailouts. Only then, [Lacker] argues, will market participants believe bailouts are over and begin to discipline financial companies themselves."

Credibly restricting the actions of future policy makers is difficult, of course, and it is not clear to what extent a strict no-bailouts policy is feasible. Nevertheless, many current reform efforts have embraced the view that such restrictions are desirable and should be pursued where possible. A leading example is the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010, which aims "to promote financial stability . . . [and] to protect the American taxpayer by ending bailouts." The Act prohibits several types of policy interventions that were used during the recent crisis by, for example, placing new restrictions on the ability of the Federal Reserve to lend funds and on the ability of the U.S. Treasury and the Federal Deposit Insurance Corporation to guarantee the debt of financial institutions. Moreover, the Act specifically states that "[t]axpayers shall bear no losses from the exercise of any authority under this title."

Is restricting policy makers's ability to bail out financial institutions an effective way to promote

financial stability? Would doing so – if feasible – actually be desirable? I address these questions in a model of financial intermediation based on the classic paper of Diamond and Dybvig (1983). In particular, I study an environment with idiosyncratic liquidity risk and limited commitment, as in Ennis and Keister (2009a). Intermediaries perform maturity transformation, which leaves them illiquid and potentially susceptible to a self-fulfilling run by investors. I introduce fiscal policy by adding a public good whose production is financed through lump-sum taxes. This approach yields a tractable model in which bailouts affect the incentives of intermediaries and their investors and these agents' actions collectively determine the susceptibility of the financial system to a crisis.

I begin the analysis by studying equilibrium outcomes in this environment when there are no restrictions on the bailout policy. If a crisis occurs, the policy maker will respond by diverting some tax revenue away from the production of public goods, using these resources to instead support the private consumption of investors facing losses. These “bailout” payments aim to improve the allocation of the remaining resources in the economy, but have undesirable effects on ex ante incentives. In particular, bailouts reduce the incentive for intermediaries to privately provision for a crisis, which leads them to become more illiquid by increasing their short-term liabilities. I characterize the conditions under which the financial system is fragile in the sense that a self-fulfilling run can occur in equilibrium and show that fragility increases as the incentive distortions associated with bailouts become more severe.

Adopting a no-bailouts policy in this environment would lead intermediaries to become more cautious and less illiquid, as emphasized by proponents of such policies. However, it would also increase the losses faced by investors who are late to withdraw in the event of a crisis. I show that a no-bailouts policy is unambiguously bad when the probability of a crisis is sufficiently small: it lowers equilibrium welfare without improving financial stability. For higher probabilities of a crisis, a no-bailouts policy may or may not be preferable, depending on parameter values. Interestingly, some economies that are not fragile when bailouts are allowed *become* susceptible to a self-fulfilling run if a no-bailouts policy is adopted.

The idea that a no-bailouts policy can make the financial system more fragile runs counter to conventional wisdom, but the mechanism behind this result is fairly intuitive. Bailouts provide insurance: they reduce the loss an investor faces if she does not withdraw her funds and a crisis occurs. Removing this insurance raises each individual's incentive to withdraw early if she expects others to do so, which makes the financial system more susceptible to a run. This argument is

familiar in the context of retail banking, where government-sponsored deposit insurance programs are explicitly designed to promote stability by limiting depositors' incentive to withdraw. Despite this similarity, discussion of this insurance role of bailouts and its effect on investor withdrawal behavior has been largely absent in both the existing literature and the current policy debate.

A good policy arrangement in the environment studied here would permit bailouts to occur, so that investors benefit from the efficient level of insurance, while offsetting the negative effects on ex ante incentives. To demonstrate this fact, I allow the policy maker to levy a Pigouvian tax on intermediaries' short-term liabilities. I show that, in addition to improving the allocation of resources, this policy has a *macroprudential* component: it decreases financial fragility relative to either of the previous policy regimes. The results here thus argue for a shift in the focus of regulatory reform away from attempts to make future policy makers behave "tough" in times of crisis and toward developing more effective policy tools for correcting distorted incentives.

In the next section, I present the model and discuss its key assumptions. In Section 3, I analyze the behavior of agents in the model (investors, intermediaries, and the policy maker) and study equilibrium outcomes when bailouts are allowed. In Section 4, I provide the corresponding analysis under a no-bailouts policy and show how adopting such a policy can, in some cases, increase fragility and lower welfare. I also discuss how the results here relate to other papers in the growing literature on the effects of financial-sector bailouts and optimal regulatory policy in the presence of limited commitment. Finally, I show how combining bailouts with a tax on short-term liabilities improves outcomes relative to either of these policy regimes in Section 5 and offer some concluding remarks in Section 6.

## 2 The Model

The analysis is based on a version of the Diamond and Dybvig (1983) model augmented to include limited commitment and a public good. This section describes the basic elements of the model and defines financial fragility in this setting.

### 2.1 The environment

There are three time periods,  $t = 0, 1, 2$ , and a continuum of investors, indexed by  $i \in [0, 1]$ . Each investor has preferences given by

$$U(c_1, c_2, g; \omega_i) = u(c_1 + \omega_i c_2) + v(g),$$

where  $c_t$  is consumption of the private good in period  $t$  and  $g$  is the level of public good, which is provided in period 1. The functions  $u$  and  $v$  are assumed to be strictly increasing, strictly concave, and to satisfy the usual Inada conditions. In addition, the coefficient of relative risk aversion for the function  $u$  is assumed to be constant and greater than one. The parameter  $\omega_i$  is a binomial random variable with support  $\Omega_i \equiv \{0, 1\}$  whose value is realized in period 1 and is private information. If  $\omega_i = 0$ , investor  $i$  is said to be *impatient*; if  $\omega_i = 1$ , she is *patient*. Let  $\pi$  denote the probability with which each individual investor will be impatient. By a law of large numbers,  $\pi$  is also the fraction of investors in the population who will be impatient.

Each investor is endowed with one unit of the private good in period 0. There is a single, constant-returns-to-scale technology for transforming this endowment into private consumption in the later periods. A unit of the good invested in period 0 yields  $R > 1$  units in period 2, but only one unit in period 1. This investment technology is operated in a central location, where investors can pool resources in an intermediation technology to insure individual liquidity risk. Investors are isolated from each other in periods 1 and 2 and no trade can occur among them. Upon learning her preference type, each investor chooses either to contact the intermediation technology in period 1 and withdraw funds or to wait and withdraw in period 2. There is also a technology for transforming units of the private good one-for-one into units of the public good. This technology is operated in period 1, using goods that were placed into the investment technology in period 0.

Investors who choose to withdraw in period 1 arrive one at a time to the central location and must consume immediately upon arrival. As in Wallace (1988, 1990), this sequential-service constraint implies that the payment made to an investor can depend only on the information received by the intermediation technology up to that point, including the number of withdrawals that have taken place so far and, possibly, the aggregate state of the economy. Withdrawing investors arrive at the central location in the order given by their index  $i$ . In other words, investor  $i = 0$  knows that she has the opportunity to be the first investor to withdraw in period 1, and investor  $i = 1$  knows his withdrawal opportunity in period 1 will be the last. An investor's position in the order is private information and her action is only observable when she chooses to withdraw.<sup>1</sup>

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<sup>1</sup> In Diamond and Dybvig (1983), investors first decide when to withdraw and then are randomly assigned positions in the withdrawal order. It seems natural, however, to suppose investors have some information about their position in this order before they decide; an investor may realize, for example, that she is relatively late to act and many withdrawals have already been made. Green and Lin (2003) showed how an investor's optimal choice can depend crucially on this information. Allowing investors to know their precise position in the order – as in Green and Lin (2000), Andolfatto *et al.* (2007) and Ennis and Keister (2009b) – is a tractable way of including these effects in the analysis. The precise approach here follows Ennis and Keister (2010).

Welfare is measured by the equal-weighted average of investors' expected utilities,

$$W = \int_0^1 E [U (c_1 (i) , c_2 (i) , g; \omega_i)] di.$$

We can think of investors as being assigned their index  $i$  randomly at the beginning of period 0, in which case this expression measures the expected utility of each investor ex ante, before any individual-specific characteristics are revealed.

## 2.2 The decentralized economy

The intermediation technology is operated by a large number of identical intermediaries. Each correctly anticipates that a fraction  $\pi$  of its investors will be impatient and behaves competitively in the sense that it considers its own effect on economy-wide resource constraints to be negligible. Intermediaries act to maximize the expected utility of their investors at all times. However, as in Ennis and Keister (2009a, 2010), they cannot commit to future actions; the payment made to each investor is always determined as a best response to the current situation when she withdraws. The public good is provided by a benevolent policy maker who taxes deposits in period 0 at rate  $\tau$  and places this revenue into the investment technology. In period 1, the policy maker can use these resources to produce the public good and, if a crisis is underway, to make transfer (“bailout”) payments to financial intermediaries.<sup>2</sup> The policy maker is also unable to commit to future actions; any bailout payments will be chosen as a best response when the payment is made.

## 2.3 Financial crises

Investors can potentially condition their actions on an extrinsic “sunspot” signal. Let  $S = \{\alpha, \beta\}$  be the set of possible states and  $\{1 - q, q\}$  the probabilities of these states, respectively. Investor  $i$  chooses a strategy that assigns an action to each possible realization of her preference type  $\omega_i$  and of the state,

$$y_i : \Omega_i \times S \rightarrow \{0, 1\},$$

where  $y_i = 0$  corresponds to withdrawing early and  $y_i = 1$  corresponds to waiting until period 2.

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<sup>2</sup> Notice that this type of bailout policy is entirely consistent with the sequential service constraint, since all taxes are collected before any consumption takes place. I assume the sequential service constraint applies to the policy maker as well as to the intermediaries and, hence, the approach here is not subject to the Wallace (1988) critique of Diamond and Dybvig (1983). Other papers have introduced taxation into the Diamond-Dybvig framework in a similar way; see, for example, Freeman (1988), Boyd *et al.* (2002), and Martin (2006). The goal of fiscal policy in those papers, however, is to fund a deposit insurance system rather than to pursue an independent objective like the provision of a public good.

Let  $y$  denote a profile of strategies for all investors. The state  $s$  can be thought of as representing investor sentiment; it has no fundamental impact on the economy, but in equilibrium it may be informative about the withdrawal plans of other investors.

Under all of the policy regimes considered here, the model has an equilibrium with  $y_i(\omega_i, s) = \omega_i$  for all  $i$  in both states. In other words, there is always a “good” equilibrium in which investors withdraw in period 1 only if they are impatient. Since no crisis occurs in this equilibrium, no bailout payments are made and no incentive distortions arise. As a result, this equilibrium implements the first-best allocation of resources. The question of interest is whether there exist other, inferior equilibria in which some patient investors *run* by withdrawing early in some state. Without loss of generality, I focus on strategy profiles in which a run occurs in state  $\beta$ .

**Definition 1:** *The financial system is fragile if there exists an equilibrium strategy profile with  $y_i(1, \beta) = 0$  for a positive measure of investors.*

Fragility thus captures the idea that the financial system is susceptible to a run based on shifting investor sentiment.

## 2.4 Timeline

The timing of decisions is summarized in Figure 1. In period 0, investors deposit their endowments with intermediaries and immediately exit the central location. The policy maker then collects a fraction  $\tau$  of total deposits as tax revenue. Intermediaries make no decisions in this stage, since there is a single asset (and thus no portfolio choice) and they cannot commit to any future payment scheme.

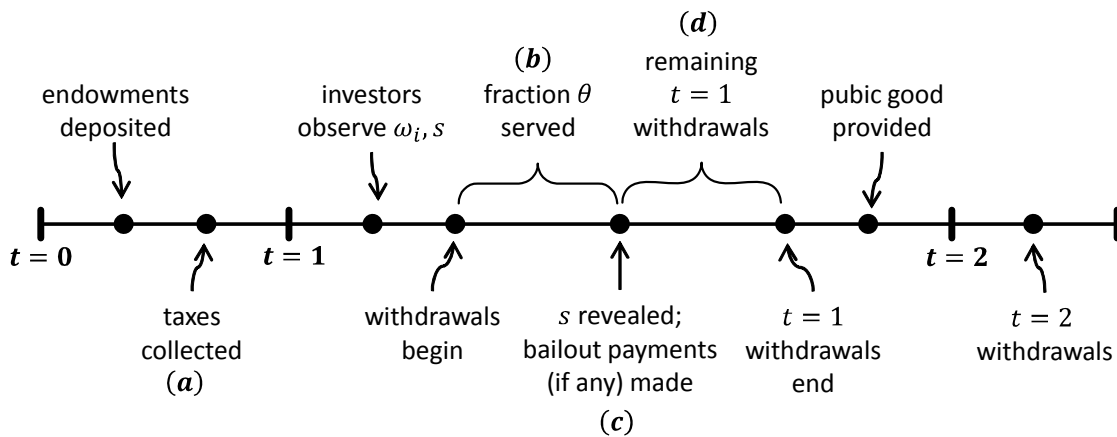


Figure 1: Timeline

At the beginning of period 1, investors are isolated from each other and from the intermediaries. After observing her own preference type  $\omega_i$  and the state  $s$ , each investor chooses whether to withdraw in period 1 or wait. Those investors who chose to withdraw in period 1 then begin to arrive at their intermediaries one at a time, in the order determined by the index  $i$ . The amount of consumption a withdrawing investor receives is determined by her intermediary as a best response to the current situation when she arrives. In particular, note that this payment is determined *after* investors have made their withdrawal decisions and thus cannot be used as a tool to influence withdrawal behavior. A withdrawing investor consumes as soon as she receives the payment from her intermediary and returns to isolation.

While investors observe the realization of  $s$  at the beginning of period 1, intermediaries and the policy maker observe the state with a lag, after a fraction  $\theta \in (0, \pi]$  of investors have withdrawn. If the state is  $\beta$  and a crisis is underway, the policy maker can choose to bail out intermediaries by transferring some tax revenue to each of them. Intermediaries combine this transfer with their own remaining funds and continue to serve withdrawing investors, making payments that are based on their updated information. The parameter  $\theta$  thus measures the speed with which both intermediaries and the policy maker are able to react to an incipient crisis.

By the end of period 1, all investors who chose to withdraw early have been served and the policy maker uses its remaining funds to provide the public good. In period 2, those investors who have not yet withdrawn contact their intermediary. Since all uncertainty has been resolved at this point and investors are risk averse, an intermediary will choose to divide its remaining resources evenly among these investors.

## 2.5 Discussion

The assumption that intermediaries and the policy maker cannot commit to plans of action before investors begin to withdraw follows Ennis and Keister (2009a) and is an essential ingredient for financial fragility in this environment. As is well known, an authority with full commitment power could eliminate the possibility of a run by promising to suspend payments as soon as it becomes clear that a run is underway (Diamond and Dybvig, 1983, Section III). This policy would guarantee a patient investor that her funds are safe regardless of the actions of other investors and thereby remove any possible incentive for her to withdraw early. However, while withdrawals are often restricted during financial crises, the policies observed in practice differ substantially from this



prescription. In particular, suspensions and other restrictions on withdrawals are typically implemented relatively late in the course of a crisis, rather than at the first sign of trouble.<sup>3</sup> Ennis and Keister (2009a) show how this type of delay arises naturally when intermediaries cannot commit and how the anticipation of such delay can give patient investors an incentive to run.

Much of the existing literature generates bank runs in environments with commitment by not allowing contracts with suspension clauses, instead assuming that investors must be given a pre-specified payout until an intermediary is completely out of funds.<sup>4</sup> While this “simple contracts” approach is often analytically convenient, it is at odds with the fact that the liabilities of financial intermediaries are routinely altered during periods of crisis. In the Argentinean crisis of 2001-2, for example, bank deposits were partially frozen, dollar-denominated deposits were forcibly converted to pesos at an unfavorable exchange rate, and some deposits were replaced with long-term bonds. Similarly, in the Cypriot banking crisis of March 2013, all deposits were partially frozen and some deposits were forcibly converted to equity as part of the bailout package funded by the European Stability Mechanism and the International Monetary Fund. The approach here, in which the payment schedule is flexible and evolves in response to the situation at hand, captures the salient features of these episodes.

The assumption that the policy maker cannot pre-commit to a detailed bailout plan is also essential. If the policy maker could commit in period 0 to make a fixed payment to each intermediary in the event of a run, it could implement a form of deposit insurance. In particular, by setting this payment large enough, the policy maker could ensure that patient investors are always better off with drawing in period 2. This insurance policy would have no direct cost in equilibrium (as in Diamond and Dybvig, 1983, Section V), since a run would never occur.<sup>5</sup> As became clear in the financial crisis of 2008, however, many deposit-like liabilities are not covered by explicit deposit insurance but may nevertheless benefit from bailouts. At the same time, explicit deposit insurance guarantees may not be fully honored in times of crisis, as demonstrated by the events in Argentina and Cyprus as well as by the dispute over the treatment of British and Dutch “Icesave” deposits

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<sup>3</sup> In discussing the U.S. banking crisis of 1933, for example, Friedman and Schwartz (1963, p.330) state that “[s]uspension occurred after, rather than before, liquidity pressures had produced a wave of bank failures without precedent.” See also Dominguez and Tesar (2007) and Ennis and Keister (2009a), who discuss the timing of the policy response to the crisis in Argentina in 2001-2.

<sup>4</sup> See, for example, Postlewaite and Vives (1987), Cooper and Ross (1998), Allen and Gale (2004), and Goldstein and Pauzner (2005), among many others.

<sup>5</sup> For some parameter values, however, there will be an indirect cost if the policy maker needs to collect extra taxes in period 0 for this commitment to be credible. In these cases, ensuring financial stability through this type of insurance would distort the allocation of resources toward the public sector.

in a failed Icelandic bank in 2008. In the model here, investors correctly anticipate that the policy maker will provide insurance, but only to the extent that it is desirable to do so ex post.<sup>6</sup>

Another essential ingredient for financial fragility in this setting is the presence of aggregate uncertainty in the form of the sunspot variable  $s$ . When intermediaries' payments to investors are chosen optimally, rather than taken as an exogenous parameter of the model, a fully-anticipated bank run cannot occur. If an intermediary expected all of its investors to withdraw early with certainty, for example, its best response would be to simply give each investor the after-tax value of her initial deposit back when she withdraws. An individual patient investor would then have no incentive to join the run; she would prefer that the intermediary keep her funds until period 2 and earn the return  $R > 1$ . For a crisis to arise in this setting, it must be the case that intermediaries and the policy maker are initially uncertain about investors' actions. In equilibrium, this uncertainty arises if the run occurs in one sunspot state and not the other. Moreover, intermediaries and the policy maker must remain uncertain about these actions while some withdrawals are being made. If intermediaries were able to observe the state before any withdrawals take place – that is, if  $\theta$  were zero – the above logic would apply as soon as the state is revealed. When  $\theta$  is positive, however, intermediaries must make payments to some investors before observing the state. Once an intermediary discovers a run is underway, it may no longer be feasible to give each remaining investor the after-tax value of her initial deposit back in period 1. Patient investors who are able to withdraw early on may then find it optimal to join the run.

Previous work using models with flexible payment schedules has assumed that intermediaries and policy makers never observe the state, but instead must infer the state from the flow of withdrawals.<sup>7</sup> In the equilibria I study below, they would be able to infer that a run is underway if the fraction of investors withdrawing goes above  $\pi$ , the fraction of impatient investors. In the special case of  $\theta = \pi$ , therefore, the approach I introduce here is equivalent to that in the existing literature. For  $\theta < \pi$ , this more general approach allows one to study how equilibrium outcomes depend on the speed of the policy reaction to an incipient crisis. I show below, for example, how the minimal value of  $\theta$  necessary to generate financial fragility varies across policy regimes.

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<sup>6</sup> In related work, Cooper and Kempf (2013) study a model with heterogeneous agents in which a policy maker without commitment makes a binary choice between providing full deposit insurance, funded by ex post taxes, and taking no action. They show that whether deposit insurance will be provided in this setting depends on how the policy redistributes wealth across agents. A run in their model is completely unanticipated, however, which implies that deposit insurance does not distort intermediaries' ex ante incentives.

<sup>7</sup> Examples include Wallace (1988, 1990), Green and Lin (2003), Peck and Shell (2003) and Ennis and Keister (2010).

### 3 Equilibrium with Bailouts

I begin the analysis by studying equilibrium when there are no restrictions on the policy maker's choice of bailout payments. I ask under what conditions the financial system is fragile in this regime and derive the equilibrium allocation of resources in these cases. To do so, I first study a particular strategy profile in which some patient investors withdraw early in state  $\beta$  but others do not. I then show that the financial system is fragile if and only if there is an equilibrium in which investors follow this particular profile.

#### 3.1 A partial-run strategy profile

Consider the following strategy profile for investors:

$$\begin{aligned} y_i(\omega_i, \alpha) &= \omega_i && \text{for all } i \text{ and} \\ y_i(\omega_i, \beta) &= \begin{cases} 0 \\ \omega_i \end{cases} && \text{for } \begin{cases} i \leq \theta \\ i > \theta \end{cases}. \end{aligned} \tag{1}$$

Under this profile, impatient investors withdraw in period 1 in both states, as must occur in any equilibrium since they only value consumption in period 1. All patient investors wait until period 2 in state  $\alpha$ , but in state  $\beta$  those patient investors who have an opportunity to withdraw relatively early in period 1 – before intermediaries and the policy maker observe the state – choose to do so.

In the subsections that follow, I derive the best responses of intermediaries and the policy maker to this strategy profile by working backward through the decisions labelled with letters in Figure 1, establishing: (d) how intermediaries allocate their remaining resources after the state is revealed and any bailouts payments have been made, (c) what bailout payments the policy maker would choose to make in the event of a crisis, (b) how the anticipation of these bailout payments affects intermediaries' incentives as the first fraction  $\theta$  of investors withdraw, and (a) what tax rate the policy maker will choose in period 0. I then study how these best responses determine investors' withdrawal incentives and the fragility of the financial system.

#### 3.2 The allocation of remaining private consumption

First, consider decision (d) for intermediary  $j$ , which arises after a fraction  $\theta$  of the intermediary's investors have withdrawn, the state has been revealed, and any bailout payments have been made. Let  $\psi_s^j$  denote the quantity of resources the intermediary has available for its remaining investors. From the strategy profile (1) and the observed state  $s$ , the intermediary can calculate the fraction

$\hat{\pi}_s$  of these investors who are impatient and will withdraw in period 1, which is either

$$\hat{\pi}_\alpha \equiv \frac{\pi - \theta}{1 - \theta} \quad \text{or} \quad \hat{\pi}_\beta \equiv \pi. \quad (2)$$

Since all uncertainty has been resolved at this point, the intermediary will choose to give a common amount  $c_{1s}^j$  to each remaining impatient investor in period 1 and a common amount  $c_{2s}^j$  to each remaining patient investor in period 2. These payments will be chosen to solve

$$V(\psi_s^j, \hat{\pi}_s) \equiv \max_{\{c_{1s}^j, c_{2s}^j\}} (1 - \theta) [\hat{\pi}_s u(c_{1s}^j) + (1 - \hat{\pi}_s) u(c_{2s}^j)] \quad (3)$$

subject to the feasibility constraint

$$(1 - \theta) \left[ \hat{\pi}_s c_{1s}^j + (1 - \hat{\pi}_s) \frac{c_{2s}^j}{R} \right] = \psi_s^j. \quad (4)$$

Letting  $\mu_s^j$  denote the multiplier on the constraint, the solution to the problem is characterized by the first-order condition

$$u'(c_{1s}^j) = R u'(c_{2s}^j) = \mu_s^j. \quad (5)$$

It is straightforward to show that the optimal values of both  $c_{1s}^j$  and  $c_{2s}^j$  are strictly decreasing functions of  $\hat{\pi}_s$ . When a larger fraction of the remaining investors are impatient, fewer resources are held in the investment technology long enough to earn the return  $R$  and the intermediary responds by allocating less consumption to each of its remaining investors.

### 3.3 Bailout policy

Next, consider decision (c) in Figure 1, which arises when the policy maker observes the state to be  $\beta$ . In this case, the policy maker knows that a run has occurred as specified in (1) and, therefore, that some of the  $\theta$  investors who have already withdrawn were actually patient. As a result, a relatively large fraction of the remaining investors –  $\hat{\pi}_\beta$  from equation (2) – are impatient and need to consume in period 1. These additional early withdrawals will lower the consumption levels  $\{c_{1s}^j, c_{2s}^j\}$  of the remaining investors as described above. The policy maker can mitigate this decline by making bailout payments  $\{b^j\}$  to intermediaries using the tax revenue collected in period 0. In making these payments, the policy maker will aim to efficiently divide the economy's remaining resources between public and private consumption, recognizing that each intermediary will allocate whatever resources it has available according to (3). Let  $\sigma(j)$  denote the distribution

of investors across intermediaries, so that the total size of the bailout package is given by

$$b \equiv \int b^j d\sigma(j).$$

The policy maker will choose the bailout payments to solve

$$\max_{\{b^j\}} \int V(\psi_\beta^j; \hat{\pi}_\beta) d\sigma(j) + v(\tau - b)$$

subject the resource constraints

$$\psi_\beta^j = 1 - \tau - \theta c_1^j + b^j \quad \text{for all } j,$$

which simply state that the resources available to intermediary  $j$  equal the initial deposit minus the tax  $\tau$  and whatever payment  $c_1^j$  was made to the fraction  $\theta$  of investors who have already withdrawn, plus the bailout payment  $b^j$ .

The first-order condition for this problem requires

$$v'(\tau - b) = \mu_\beta^j \quad \text{for all } j, \tag{6}$$

which says that the policy maker will equate the marginal value of public consumption in state  $\beta$  to the marginal value of private consumption in that state for the remaining investors *in each intermediary*. In other words, the solution to this problem must equalize the marginal value of resources  $\mu_\beta^j$  across all intermediaries. For a given size of the total bailout package  $b$  per investor, this entails setting

$$b^j = b + \theta [c_1^j - \bar{c}_1] \quad \text{for all } j, \tag{7}$$

where  $\bar{c}_1 \equiv \int c_1^j d\sigma(j)$  is the economy-wide average of the payment  $c_1^j$  given to the fraction  $\theta$  of investors who have already withdrawn. The remaining resources  $\psi_\beta^j$  available to intermediary  $j$  will then depend only on aggregate conditions,

$$\psi_\beta^j = 1 - \tau - \theta \bar{c}_1 + b, \tag{8}$$

which the intermediary takes as given, and not on its own choice of  $c_1^j$ .

The incentive problems created by this policy are clear: an intermediary with fewer remaining resources (because it set  $c_1^j$  higher) will receive a larger bailout payment. This larger payment implies that fewer funds are available for making transfers to other intermediaries and for public

consumption. In equilibrium, of course, all intermediaries will choose the same value of  $c_1^j$  and receive the same bailout payment  $b^j$  per investor, but these values will be higher than is socially desirable because of the external effect each intermediary's choice has in state  $\beta$ .<sup>8</sup>

### 3.4 Distorted incentives

Moving to decision (b) in Figure 1, intermediary  $j$  must decide how much consumption to give to each of the fraction  $\theta$  of investors who withdraw before it observes the state. Since no information is revealed as these withdrawals take place, the intermediary will choose to give the same amount  $c_1^j$  to each of them. The value of  $c_1^j$  will be chosen to maximize

$$\theta u(c_1^j) + (1 - q) V(1 - \tau - \theta c_1^j; \hat{\pi}_\alpha) + q V(1 - \tau - \theta \bar{c}_1 + b; \hat{\pi}_\beta). \quad (9)$$

The first term corresponds to the fraction  $\theta$  of the intermediary's investors who will receive the chosen amount  $c_1^j$ . The second and third terms represent the expected utility from distributing the intermediary's remaining resources in each state – including the bailout payment  $b^j$  in state  $\beta$  – according to the efficient plan from (3).

The first-order condition for this problem is

$$\begin{aligned} u'(c_1^j) &= (1 - q) V'(1 - \tau - \theta c_1^j; \hat{\pi}_\alpha) \\ &= (1 - q) \mu_\alpha^j. \end{aligned} \quad (10)$$

This condition highlights the incentive distortion described above: the intermediary chooses  $c_1^j$  to equate the marginal value of resources before the state is known to the expected shadow value of resources *in the no-run state*, ignoring the value of resources in the event of a run. This behavior is individually optimal because the bailout policy (7) implies that the resources available to the intermediary in the run state will be independent of its own choice of  $c_1^j$ .

Condition (10) implicitly defines the intermediary's optimal choice of  $c_1^j$  as a function of the tax rate  $\tau$ . Since all intermediaries face the same decision problem, I omit the index  $j$  and denote this function by  $c_1(\tau)$ . Note that  $c_1(\tau)$  does not depend on the choices of other intermediaries, as summarized by  $\bar{c}_1$ , nor on the size of the bailout package  $b$ . While these variables are important

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<sup>8</sup> Note that, in principle, a similar incentive problem could arise in state  $\alpha$  if the policy maker made bailout payments to intermediaries that chose an unusually high level of  $c_1^j$  in that state as well. I assume that bailout payments are only made in the event of a financial crisis. This assumption could be justified by reputation concerns, which will be significant for decisions made in normal times but much less important for a policy maker facing a rare event like a financial crisis.

determinants of investors' consumption in state  $\beta$ , condition (10) shows that they do not affect an intermediary's optimal choice at decision (b) in the figure.

Using (5) and (10), we see that the solution to this problem will satisfy  $c_1 < c_{2\alpha}$  as long as

$$q < \frac{R-1}{R}. \quad (11)$$

If this inequality were reversed, a patient investor would prefer to withdraw early even if all other patient investors chose to wait. In such cases, the profile in (1) is not consistent with equilibrium, but other, more complex partial-run strategy profiles may be. To avoid these complications, I restrict attention to the case (11), where the probability of a crisis is sufficiently small, throughout the analysis.

### 3.5 Choosing the tax rate

In decision (a), the policy maker will choose the tax rate  $\tau$  to maximize ex ante welfare,

$$\theta u(c_1(\tau)) + (1-q)[V(\psi_\alpha; \hat{\pi}_\alpha) + v(g_\alpha)] + q[V(\psi_\beta; \hat{\pi}_\beta) + v(g_\beta)], \quad (12)$$

subject to the resource constraints

$$\psi_\alpha = 1 - \tau - \theta c_1(\tau) \quad \text{and} \quad (13)$$

$$\psi_\beta = 1 - \tau - \theta c_1(\tau) + b(\tau), \quad (14)$$

and the government budget constraint in each state,

$$g_\alpha = \tau \quad \text{and} \quad g_\beta = \tau - b(\tau), \quad (15)$$

where the function  $c_1(\tau)$  is implicitly defined by (10) and the function  $b(\tau)$  by (6). The first-order condition for this problem is

$$\begin{aligned} (u'(c_1(\tau)) - (1-q)\mu_\alpha - q\mu_\beta) \theta \frac{dc_1}{d\tau} &= (1-q)[\mu_\alpha - v'(\tau)] \\ &+ q[\mu_\beta - v'(\tau - b)] \left(1 - \frac{db}{d\tau}\right). \end{aligned} \quad (16)$$

Note that (6) implies that the term in square brackets on the second line of this equation is zero. Because the size of the bailout package will be chosen to equate the marginal values of public and private consumption in state  $\beta$ , the first-order effect of changes in  $b$  on welfare is zero. Using this

fact and (10), the first-order condition simplifies to

$$v'(\tau) = \mu_\alpha + \frac{q}{1-q} \mu_\beta \theta \frac{dc_1}{d\tau}. \quad (17)$$

If the probability of a crisis  $q$  were zero, the tax rate would be set to equate the marginal value of the public good in state  $\alpha$  with the corresponding marginal value of private consumption, which is the standard Samuelson condition for the efficient provision of a public good. When  $q$  is positive, the policy maker recognizes that  $\tau$  affects intermediaries' choice of  $c_1$  and, hence, can be used to partially correct the incentive distortion caused by the bailout policy.

### 3.6 Equilibrium and Fragility

The above analysis establishes that the best responses of intermediaries and the policy maker to the strategy profile (1) under the policy regime with bailouts can be summarized by a vector

$$\mathbf{c}^B \equiv \left\{ c_1^B, \{c_{1s}^B, c_{2s}^B, g_s^B\}_{s=\alpha, \beta} \right\}$$

that specifies the private consumption levels of each investor and the level of public consumption in each state. The components of this vector are jointly determined by equations (4) - (6), (10), (13) - (15), and (17).

When does the economy have an equilibrium in which investors follow the strategy profile (1)? To answer this question, we must compare the payoff each investor would receive from the allocation  $\mathbf{c}^B$  if she withdraws early to that she would receive if she waits. An impatient investor will always strictly prefer to withdraw early, as specified in the profile, so we only need to consider the actions of patient investors. Condition (5) implies that  $c_{1s}^B < c_{2s}^B$  holds for any value of  $\hat{\pi}_s$  and, hence, a patient investor with  $i > \theta$  will prefer to wait until period 2 to withdraw in both states.<sup>9</sup> As mentioned above, condition (11) implies that  $c_1^B < c_{2\alpha}^B$  always holds, so that a patient investor with  $i \leq \theta$  will strictly prefer to wait in state  $\alpha$ . The only question, therefore, is whether a patient investor with  $i \leq \theta$  has an incentive to join the run in state  $\beta$ . If she does, she will arrive before the state is revealed and receive  $c_1^B$ . If she deviates by waiting until period 2, she will receive  $c_{2\beta}^B$ . We can, therefore, construct an equilibrium in which investors follow (1) if and only if the allocation

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<sup>9</sup> This result implies that a run in this model is necessarily partial. Once an intermediary observes the state, its reaction will be such that the remaining patient investors have no incentive to withdraw early. See Ennis and Keister (2010) for a related model in which an intermediary never observes the state, but is able to gradually make inferences about it from the flow of withdrawals. In that setting, a run must occur in waves, with only some investors withdrawing in each wave.



$\mathbf{c}^B$  satisfies

$$c_1^B \geq c_{2\beta}^B. \quad (18)$$

Recall, however, that the payoffs  $c_1^B$  and  $c_{2\beta}^B$  were calculated under the assumption that investors follow the strategy profile (1), which is only one of many possible ways in which a partial run could be organized. When condition (18) is violated, could an equilibrium exist in which a different set of investors choose to run in state  $\beta$  and/or some investors run in state  $\alpha$  as well? The following proposition shows that the answer is negative; (18) is also a *necessary* condition for the financial system to be fragile. Focusing on the particular strategy profile (1) is thus without loss of generality: if this strategy profile is not part of an equilibrium, then there is no equilibrium in which some investors run in state  $\beta$ . Proofs of all results are given in the appendix unless otherwise noted.

**Proposition 1** *The financial system is fragile under the bailouts regime if and only if (18) holds.*

In comparing outcomes across different policy regimes, it will be useful to study the set of all economies that are fragile under a given regime. An economy is characterized by the parameters  $e \equiv (R, \pi, u, v, \theta, q)$ . Let  $\Phi^B$  denote the set of economies that are fragile under the bailouts regime. Proposition 1 establishes that  $e \in \Phi^B$  if and only if (18) holds, in which case  $\mathbf{c}^B$  represents the allocation of resources in the run equilibrium.

Figure 2 depicts the set  $\Phi^B$  as the parameters  $q$  and  $\theta$  are varied using the utility functions

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad \text{and} \quad v(g) = \delta \frac{g^{1-\gamma}}{1-\gamma}$$

with three different values of  $\delta$ . The other parameter values are given by  $(R, \pi, \gamma) = (1.05, 1/2, 8)$ . Two general patterns stand out in all panels of the figure. First, for a given value of  $q$ , the delay parameter  $\theta$  must be sufficiently large for the financial system to be fragile. When  $\theta$  is small, intermediaries are able to react quickly to an incipient run by adjusting payments to the appropriate level. This quick reaction ensures that the losses created by the run are small and, as a result, the payoff to a patient investor from waiting to withdraw,  $c_{2\beta}$ , remains higher than the payoff from joining the run,  $c_1$ . Second, the threshold value of  $\theta$  decreases with  $q$ . As a crisis becomes more likely, intermediaries have an incentive to give a higher return to the first fraction  $\theta$  of investors who withdraw because there is no (private) cost of having done so if the crisis state occurs, as shown in (8). This higher return makes withdrawing early more attractive to a patient investor,

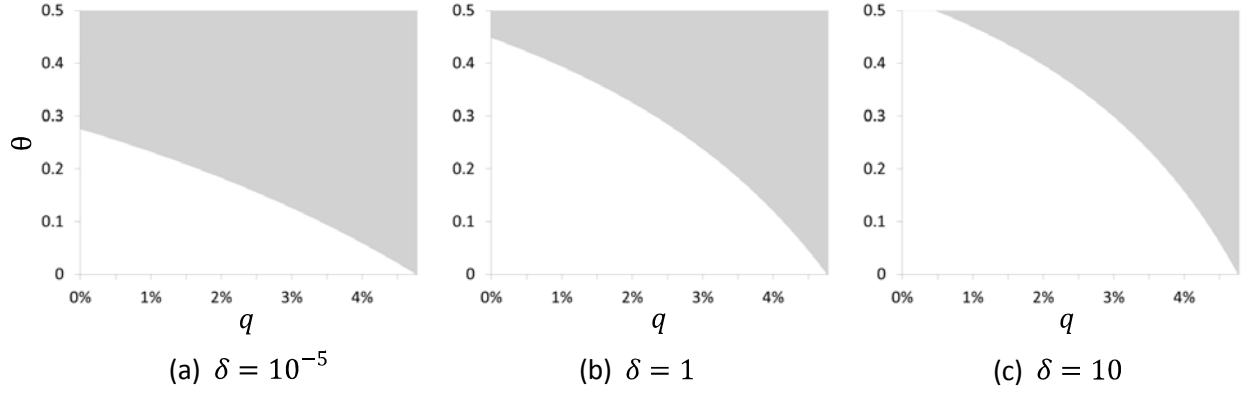


Figure 2: The fragile set  $\Phi^B$  under the bailouts regime

which makes the financial system fragile for lower values of  $\theta$ . As  $q$  approaches the bound in (11), this threshold drops to zero: even if the intermediary is able to react almost immediately to a run, the first few patient investors will still have an incentive to withdraw early.

The parameter  $\delta$  in this example measures the relative importance of the public good and is a key determinant of the tax rate  $\tau$ . When  $\delta$  is small, as in panel (a) of the figure, the policy maker collects relatively little tax revenue in period 0 and the bailout payments  $b^j$  made in state  $\beta$  are correspondingly small. As  $\delta$  increases, the policy maker collects more tax revenue in order to provide more of the public good. If a crisis occurs, the bailout payments determined by equation (6) are then larger, which raises the private consumption levels  $(c_{1\beta}, c_{2\beta})$  and makes withdrawing early less attractive to a patient investor. For this reason, the set  $\Phi^B$  shrinks as we move to panels (b) and (c). This effect is particularly pronounced for lower values of the probability  $q$ , where the incentive distortions associated with bailouts are small.

### 3.7 Discussion

It is worth noting that there is no role for liquidity facilities or other forms of lending to the financial sector in this model, even though a crisis is driven by self-fulfilling beliefs. Once intermediaries and the policy maker realize a run is underway, the problem is no longer one of illiquidity in the financial system because all future payments will be adjusted based on this information. Instead, the problem is that some patient investors have already withdrawn, creating a misallocation of resources that shrinks the set of feasible consumption levels  $(c_{1s}, c_{2s})$  for the remaining investors. The only way to mitigate the losses suffered by these investors is through real transfers from the public sector.

The model also highlights a key conceptual difference between bailouts and deposit insurance programs. Deposit insurance is an ex ante promise that aims to influence withdrawal behavior by assuring investors that they will not suffer losses. A bailout, in contrast, is an ex post response that aims to mitigate the effects of a crisis. The following proposition shows that the policy maker's best response to a crisis is never to provide full deposit insurance; investors who are late to withdraw always suffer losses in the crisis state.

**Proposition 2** *For any  $e \in \Phi^B$ , we have  $(c_{1\beta}^B, c_{2\beta}^B, g_\beta^B) \ll (c_{1\alpha}^B, c_{2\alpha}^B, g_\alpha^B)$ .*

Fully insuring deposits would shift all of the losses associated with a crisis onto the public sector, raising the marginal value of public consumption in state  $\beta$  above investors' marginal utility of private consumption. Instead, the best-response bailout policy in (6) divides the losses between the public and private sectors, which implies that investors' funds in the financial system are only partially insured.

In the sections that follow, I analyze two policy measures that aim to mitigate the incentive problem associated with bailouts and improve welfare, beginning with a prohibition on bailouts.

## 4 Equilibrium under a No-Bailouts Policy

Suppose a law is passed requiring the policy maker to set  $b^j = 0$  for all intermediaries in all states of nature. As described in the introduction, the idea that eliminating bailouts is an effective way to discourage risky behavior and thereby promote financial stability has figured prominently in recent policy debates. What effects would such a law have in this model?

The first step in answering this question is to derive the best responses of intermediaries and the policy maker to a run under this new regime by again working backward through the labelled decisions in Figure 1. Decision (d) is unchanged: intermediaries will still allocate their remaining resources according to (3). Decision (c) is now trivial: the policy maker is required to set  $b^j = 0$  for all  $j$ . The analysis for this case begins, therefore, with decision (b), in which intermediaries choose how much consumption to give to each of the fraction  $\theta$  of their investors who withdraw before the state is revealed.

#### 4.1 Corrected incentives ...

Under a no-bailouts regime, each intermediary must use its own resources to provide consumption to all of its investors in both states. When faced with decision (b) in Figure 1, intermediary  $j$  will now choose  $c_1^j$  to solve

$$\max_{\{c_1^j\}} \theta u(c_1^j) + (1-q)V(1-\tau-\theta c_1^j; \hat{\pi}_\alpha) + qV(1-\tau-\theta c_1^j; \hat{\pi}_\beta).$$

Note the crucial difference between this expression and (9): the resources available to intermediary  $j$  after  $\theta$  withdrawals is now the same in both states. The first-order condition for this problem is

$$u'(c_1) = (1-q)\mu_\alpha + q\mu_\beta. \quad (19)$$

Comparing (19) with (10) shows how the no-bailout policy changes incentives. An intermediary now equates the marginal value of resources before it observes the state to the expected future value of resources taking into account *both* possible outcomes.

In decision (a) in Figure 1, the policy maker will again choose the tax rate  $\tau$  to maximize (12) subject to the resource constraints (13) and (14), but with the function  $c_1(\tau)$  now defined by (19) and with  $b(\tau)$  set to zero, which implies

$$g_\alpha = g_\beta = \tau. \quad (20)$$

The first-order condition is again given by (16), but with both  $b$  and the derivative  $db/d\tau$  set to zero. Note that (19) implies that the left-hand side of this equation is now zero. When intermediaries choose  $c_1$  taking into account the marginal value of resources in both possible states, the first-order effect of changes in  $c_1$  on welfare are zero. Using this fact, condition (16) simplifies to

$$v'(\tau) = (1-q)\mu_\alpha + q\mu_\beta. \quad (21)$$

Because all tax revenue must now go into the public good in both states, the policy maker sets the marginal value of public consumption equal to the expected marginal value of private consumption.

Let  $\mathbf{c}^N$  denote the allocation vector characterized by equations (4), (5), (13), (14) with  $b(\tau) = 0$ , and (19) - (21). This allocation represents the best responses of intermediaries and the policy maker to the strategy profile (1) under a no-bailouts policy. As in the previous section, comparing two elements of this vector yields a necessary and sufficient condition for the existence of *any* equilibrium in which some investors choose to run in state  $\beta$ .

**Proposition 3** *The financial system is fragile under the no-bailouts regime if and only if  $c_1^N \geq c_{2\beta}^N$  holds.*

The proof, which follows that of Proposition 1 closely, is omitted. Let  $\Phi^N$  denote the set of economies  $e$  for which this condition holds.

## 4.2 ... but competing effects on fragility

Moving from the bailouts regime to a no-bailouts policy has two competing effects on financial fragility. First, intermediaries become more cautious when dealing with the first  $\theta$  investors to withdraw. To make this statement precise, define the *degree of illiquidity* in the financial system to be

$$\rho \equiv \frac{\theta c_1}{1 - \tau}. \quad (22)$$

Since a fraction  $\theta$  of investors will withdraw  $c_1$  before intermediaries or the policy maker are able to react to an incipient crisis, the numerator of this expression has a natural interpretation as the total short-term liabilities of the financial system. The denominator is the period-1 value of intermediaries' total assets. In other words,  $\rho$  is the ratio of the short-term liabilities of the financial system to the short-run value of its assets, a natural measure of illiquidity. The following proposition shows that a no-bailouts policy reduces illiquidity in the financial system.

**Proposition 4**  $\rho^N < \rho^B$  holds for all  $q > 0$ .

This result captures the popular idea that, by eliminating the incentive distortions associated with bailouts, a no-bailouts policy will lead intermediaries to hold more resources in reserve as a provision against bad outcomes. These actions lower the return earned by investors who withdraw early and thus tend to reduce the incentive for patient investors to run.

However, prohibiting bailouts has another effect on investors' withdrawal incentives. For a given value of  $c_1$ , a crisis now leads to lower consumption levels  $(c_{1\beta}, c_{2\beta})$  for the remaining investors because no public funds are available to mitigate their losses. These lower payoffs tend to increase the incentive for those patient investors with  $i \leq \theta$  to withdraw early, while the payment  $c_1$  is still available.

The net effect of a no-bailouts policy on financial fragility is thus ambiguous: while it leads intermediaries to become more cautious, it also makes some investors' payoffs more sensitive to

the withdrawal behavior of others. The next proposition shows that either of these two effects can dominate, depending on parameter values.

**Proposition 5** *There exist economies in  $\Phi^N$  that are not in  $\Phi^B$  and vice versa.*

This result is demonstrated in Figure 3, which adds the set  $\Phi^N$  to Figure 2. Focus first on panel (b) in the figure. When  $q$  is small, the threshold value of  $\theta$  for fragility to arise is much lower under a no-bailouts policy. Economies in the medium-shaded region on the left side of the graph are not fragile under the bailouts regime but *become* fragile when a no-bailouts policy is adopted. To understand why, note that when a crisis is unlikely, the incentive distortion associated with the bailouts regime is small and, as a result, the initial payment on early withdrawals ( $c_1$ ) is similar under the two different regimes. However, the payoff from staying invested in the event of a run ( $c_{2\beta}$ ) is much lower under a no-bailouts policy because intermediaries receive no transfer from the public sector. As a result, patient investors have a stronger incentive to withdraw early in state  $\beta$  under the no-bailouts regime, which translates into a lower threshold value for  $\theta$ . The magnitude of this effect depends on the size of the bailout payments  $b^j$ , which in turn depends on the tax revenue collected by the policy maker in period 0. When the parameter  $\delta$  is close to zero, as in panel (a), bailout payments are small and the threshold value of  $\theta$  is only slightly lower under a no-bailouts policy. When  $\delta$  is large, as in panel (c), bailouts payments are much larger, which implies that eliminating these bailouts lowers the threshold for fragility substantially.

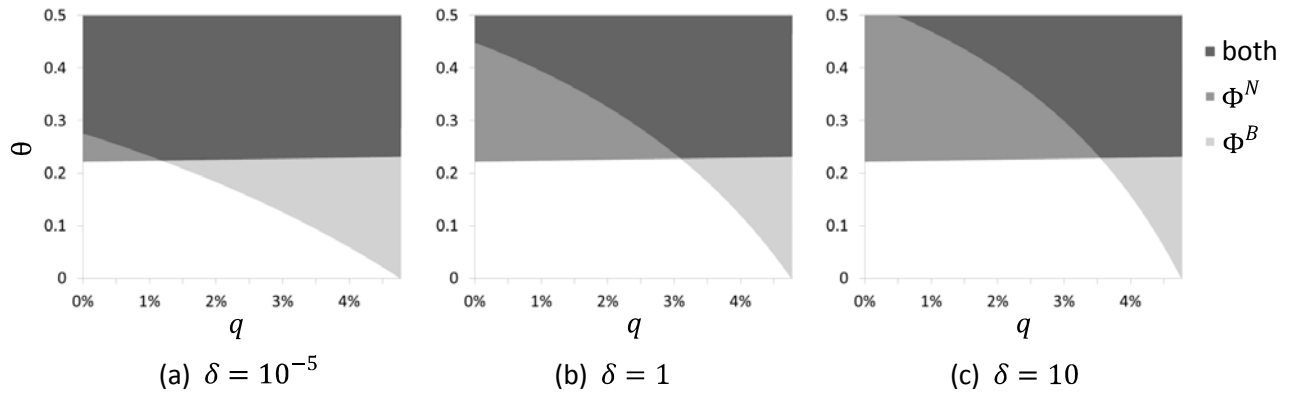


Figure 3: Comparing the sets  $\Phi^B$  and  $\Phi^N$

As the probability of a crisis increases, the incentive distortion that causes the threshold value for  $\theta$  to decrease under the bailouts regime no longer appears. Instead, intermediaries operating under a no-bailouts policy become more cautious as  $q$  increases and the threshold actually increases slightly. Moreover, the set  $\Phi^N$  is independent of  $\delta$  and thus the same in all three panels of the figure; the size of the public sector has no effect on investors' withdrawal incentives when bailout payments are prohibited. As shown in each panel, the no-bailouts policy will generate a higher threshold for fragility when the probability of a crisis is large enough (above  $\sim 3\%$  in panel (b), for example). For economies in the lightly-shaded region in the lower right-hand corner of each panel, a no-bailouts policy eliminates the run equilibrium and thus promotes financial stability.

To summarize, Figure 3 illustrates the two competing effects of a no-bailouts policy on financial stability. First, the policy leads intermediaries to become more liquid, which by itself would tend to raise the threshold level of  $\theta$  and make the financial system more stable. The magnitude of this effect is small when  $q$  is close to zero, but becomes large as  $q$  increases. Second, the no-bailouts policy makes investors more anxious to withdraw in state  $\beta$  because their payoffs are more sensitive to the withdrawal behavior of others. The magnitude of this effect is largely independent of  $q$ , since withdrawal decisions are made after investors have observed the realized state. In each panel of the figure, the first effect dominates when  $q$  is large enough, while the second effect dominates when  $q$  is smaller. Comparing across panels shows that the second effect becomes relatively more important when the public sector is larger.

### 4.3 Welfare comparison

When the financial system is fragile under one policy regime but not the other, the optimal policy is clear: selecting the non-fragile regime will ensure that no run occurs and the first-best allocation of resources obtains in both states. If the financial system is fragile under both policy regimes, the policy maker must compare the welfare levels  $W^B$  and  $W^N$  associated with the run equilibrium allocation in each case. In general, welfare may be higher under either regime, depending on parameter values. Once again, two competing effects are at work. Bailouts distort incentives and lead intermediaries to allocate too much consumption to the first fraction  $\theta$  of investors who withdraw. Prohibiting bailouts removes this distortion, but also prevents socially-desirable risk sharing between public and private consumption. The next proposition shows that when  $q$  is small enough, the second effect always has a larger impact on welfare.

**Proposition 6** *For any  $(R, \pi, \theta, u, v)$ , there exists  $\bar{q} > 0$  such that  $q < \bar{q}$  and  $e \in \Phi^B$  implies both  $e \in \Phi^N$  and  $W^N < W^B$ .*

In other words, when the probability of a crisis is sufficiently low, adopting a no-bailouts policy does not improve financial stability and necessarily lowers welfare.

Figure 4 plots the welfare gain from adopting a no-bailouts policy as a function of the probability of state  $\beta$  for the same parameter values used above and setting  $\theta = \delta = 1/2$ . When  $q = 0$ , the crisis state never occurs and both policies deliver the first-best allocation; the benefit of switching regimes is zero. As  $q$  increases, the no-bailouts regime initially lowers welfare, as established in Proposition 6. For higher values of  $q$ , however, the effects of the incentive distortion caused by bailouts become more pronounced and the pattern begins to reverse. Beyond approximately  $q = 3.5\%$ , adopting a no-bailouts policy would raise welfare in this economy.

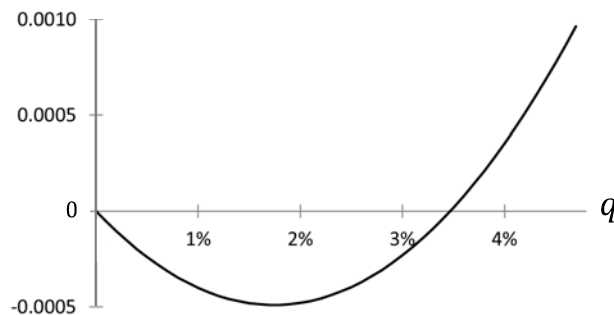


Figure 4: Welfare gain from a no-bailouts policy

#### 4.4 Relation to the literature

Propositions 5 and 6 demonstrate that prohibiting bailouts is at best an imperfect solution to the problems associated with distorted incentives and is sometimes worse than the distortions it aims to correct. This result runs counter the conclusions drawn in much of the existing literature.<sup>10</sup> Chari and Kehoe (2015), for example, study an environment where the threat of costly bankruptcy proceedings is useful for encouraging firm managers to exert an appropriate level of effort. Once

<sup>10</sup> The literature on financial-sector bailouts continues to grow rapidly and is too large to survey here. In addition to the papers discussed in this section, recent contributions include Niepmann and Schmidt-Eisenlohr (2013), who examine the strategic interaction between governments when bailouts have international spillover effects, Ranciere and Tornell (2015), who show how the anticipation of a bailout can lead to welfare-reducing financial innovation, and Nosal and Ordoñez (2013), who study how uncertainty about the government's information set can mitigate the moral hazard associated with bailouts.



outcomes are realized, however, a benevolent policy maker would prefer to avoid these costs by bailing out any firm facing bankruptcy. The anticipation of such a bailout undermines managers' ex ante incentives and thereby leads to an inferior outcome. In their setting, a strict no-bailouts policy would implement the constrained-efficient allocation of resources and, hence, clearly be desirable. They go on to study whether regulation of private contracts can be welfare improving when such commitment is infeasible. A key difference in the setting here is that bailouts represent a transfer between public and private consumption, as opposed to simply a redistribution of resources across agents. This transfer, which cannot be generated through private contracts, represents a valuable source of risk sharing in the economy that is eliminated by a no-bailouts policy. If preferences were changed so that investors received no utility from public consumption – that is, if the utility function  $U$  did not depend on  $g$  – the risk-sharing benefit of bailouts would disappear and the environment here would more closely resemble others in the literature. A no-bailouts policy would be always be desirable in this case.<sup>11</sup>

A second important difference with the existing literature lies in how bailouts affect the potential for multiplicity of equilibrium. In Farhi and Tirole (2012), allowing bailouts creates a strategic complementarity in banks' decisions and introduces multiple equilibria. Banks in their model face refinancing risk and the central bank chooses an interest rate that affects the cost of refinancing. When other banks engage in more maturity transformation, the central bank will choose to lower interest rates further in the event of a crisis, which makes doing more maturity transformation individually optimal for each bank. Commitment a no-bailouts policy removes this complementarity and leaves a unique, efficient equilibrium in their model. In the environment here, the anticipation of a bailout also leads banks to do more maturity transformation, which by itself would increase the incentive for investors to run in state  $\beta$ . However, as described above, the insurance provided by bailouts creates an offsetting effect that weakens patient investors' incentive to withdraw early. Depending on which of these two effects dominates, a no-bailouts policy can either eliminate or introduce a run equilibrium here, as illustrated in Figure 3.

If the run equilibrium exists under both policy regimes, the desirability of prohibiting bailouts will depend on the tradeoff between improved incentives and lost risk sharing. This tradeoff is similar in spirit to that in Gale and Vives (2002), who study dollarization as a way for a central

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<sup>11</sup> See Keister and Narasiman (2014) for a formal statement of this type of result in a related model. The idea that bailouts can be part of a desirable social insurance arrangement also appears, in different forms, in Green (2010) and Bianchi (2012).

bank to commit to a no-bailouts policy. In their model, a central bank without commitment will engage in bailouts too often, undermining bank managers' incentives to exert effort. Dollarization – adopting another country's currency – improves incentives by limiting bailouts, but causes projects to be liquidated in too many states. The desirability of prohibiting bailouts thus depends on which of two competing effects dominates, much as in the environment I study here. The nature of these effects differs across models in important ways, however, leading to differing policy conclusions.

In Gale and Vives (2002), banks are restricted to offer standard demand-deposit contracts in which the payoffs owed to investors are fixed in nominal terms. By rule, a bank must liquidate its project whenever it is unable to meet these obligations. Such liquidation is often inefficient; welfare would be higher if liquidation were less frequent, but avoiding liquidation would require giving less to investors when the project's return is lower. The role of bailouts in their model is to create this type of state-contingent payoff: by raising the general price level when the return on banks' projects is low, the central bank can decrease banks' real obligations to investors and thereby allow them to avoid liquidation. In the model here, in contrast, an intermediary's payments to its investors are allowed to be fully state contingent once the state is observed; there is no scope for the policy maker to improve matters by altering the real value of existing private claims. Instead, the role of bailouts here is to allow risk-sharing between private consumption and public consumption, which requires that bailouts take the form of fiscal transfers. The costs associated with bailouts are also different in the two models. In Gale and Vives (2002), bailouts undermine the incentive for a bank's manager to exert effort, which lowers the expected return on the bank's assets. The distortion here arises on the liabilities side, as intermediaries (and their investors) prefer to have excessively-large short-term liabilities.

As a result of these differences, the two models point toward rather different policy conclusions. In the Gale-Vives model, bailouts and the associated incentive distortions would be unnecessary if private contracts could be made more state contingent. For example, requiring banks to replace some demand deposits with more contingent liabilities, such as debt that converts to equity under some conditions, could implement the constrained efficient allocation. (This policy would be an alternative to the partial-dollarization regime they study in Section VI.B.) In particular, their model suggests that with an appropriate contractual reform in place, there would be no need to place other regulations on banks to correct bailout-related distortions. In the model here, in contrast, payoffs are already state contingent and contractual reforms such as convertible debt would be ineffective.

Instead, the model suggests policies that influence intermediaries' choice of short-term liabilities may be helpful. In the next section, I study a regime in which bailouts are allowed and the policy maker takes a Pigouvian approach to correcting the resulting distortions. I show how this approach can both generate higher welfare and improve financial stability relative to either of the policy regimes studied so far.

## 5 Taxing Short-term Liabilities

Suppose now that instead of restricting bailouts, the policy maker is given an additional tool: a Pigouvian tax on intermediaries' short-term liabilities. Specifically, when each of the first fraction  $\theta$  of investors withdraws from intermediary  $j$  and receives  $c_1^j$ , policy maker collects an additional amount  $\eta c_1^j$  from the intermediary. As discussed above,  $\theta c_1^j$  can be interpreted as the value of an intermediary's short-term liabilities per investor and, hence,  $\eta$  represents the Pigouvian tax rate on these liabilities.<sup>12</sup> To facilitate comparison with the earlier cases, I make the policy revenue neutral by giving each intermediary a lump-sum transfer equal to the average fee collected per investor,

$$T = \eta \theta \bar{c}_1. \quad (23)$$

To see how this additional policy tool changes equilibrium outcomes, I again work backward through the decisions in Figure 1. Decisions (d) and (c) are the same as under the bailouts regime in Section 3, meaning that intermediaries will allocate their remaining resources to solve (3) and the policy maker will choose the bailouts payments  $\{b^j\}$  to satisfy (6). This bailout policy now implies that the resources  $\psi_\beta^j$  available to intermediary  $j$  in the run state are equal to

$$\psi_\beta^j = 1 - \tau - \theta (1 + \eta) \bar{c}_1 + T + b,$$

which, using the rebate policy in (23), reduces to expression (8) from the earlier section.

### 5.1 Modified incentives

We can write decision (b) in Figure 1 for intermediary  $j$  as the problem of choosing  $c_1^j$  to maximize

$$\theta u(c_1^j) + (1 - q) V(1 - \tau - \theta (1 + \eta) c_1^j + T; \hat{\pi}_\alpha) + q V(1 - \tau - \theta \bar{c}_1 + b; \hat{\pi}_\beta).$$

<sup>12</sup> In principle, the policy maker could choose different values of the fee  $\eta$  as different investors withdraw. However, since investors are risk averse, the policy maker's best response will always apply the same fee  $\eta$  to each of the first  $\theta$  withdrawals.

The middle term in this expression shows how each unit of resources given to the first fraction  $\theta$  of investors who withdraw now reduces the funds available for the intermediary's remaining investors in state  $\alpha$  by  $(1 + \eta)$ . The first-order condition characterizing the solution to this problem is

$$u'(c_1^j) = (1 + \eta)(1 - q)\mu_\alpha^j. \quad (24)$$

This condition implicitly defines the intermediary's optimal choice of  $c_1^j$  as a function of both the tax rate  $\tau$  on deposits and the fee  $\eta$  on short-term liabilities.

In decision (a), the policy maker chooses  $\tau$  and  $\eta$  to maximize ex ante welfare. Using the rebate policy in (23), this problem reduces to maximizing the objective function in (12) subject to the constraints (13) - (15), but with  $c_1$  and  $b$  now as functions of both  $\tau$  and  $\eta$  rather than  $\tau$  alone. The first-order condition for  $\eta$  is

$$(u'(c_1) - (1 - q)\mu_\alpha - q\mu_\beta)\theta \frac{dc_1}{d\eta} = -q[\mu_\beta - v'(\tau - b)] \frac{db}{d\eta}.$$

Because the policy maker will choose the size of the bailout package in decision (c) according to (6), the right-hand side of this expression is zero. The fee  $\eta$  will, therefore, be chosen to make the left-hand side zero, which requires

$$u'(c_1) = (1 - q)\mu_\alpha + q\mu_\beta. \quad (25)$$

Recall that the incentive distortion created by the bailout policy leads intermediaries to undervalue resources paid out before the state is revealed because they (rationally) ignore the consequences of these actions in state  $\beta$ , as shown in (10). The fee  $\eta$  increases an intermediary's cost of using these resources, as shown in (24). How much does the policy maker want to increase this cost? Until the marginal value an intermediary assigns to these resources is equal to the expected future marginal utility of consumption, as shown in (25).

The first-order condition for the choice of  $\tau$  is again given by (16). Using (6) and (25), the condition simplifies in this case to

$$v'(\tau) = \mu_\alpha. \quad (26)$$

When there is no Pigouvian tax, the policy maker uses the tax on deposits to partially offset the distortion caused by the bailout policy, as shown in (17). Now that  $\eta$  is used to correct this distortion, (26) shows that the policy maker chooses  $\tau$  according to a standard Samuelson condition, equating the marginal values of public and private consumption in the state where no bailouts occur.

Let  $\mathbf{c}^*$  denote the allocation vector characterized by equations (4) - (6), (13) - (15), (25) and (26), which summarizes the best responses of intermediaries and the policy maker to the strategy profile (1) under the Pigouvian regime. The value of the fee  $\eta$  in this allocation can be determined by combining (24) and (25) to obtain

$$\eta^* = \frac{q\mu_\beta^*}{(1-q)\mu_\alpha^*}. \quad (27)$$

If the probability of a crisis were zero, no incentive distortion would arise and the policy maker would set  $\eta$  to zero. Higher values of  $q$  create larger distortions, which leads the policy maker to choose a higher fee. Note that the objective of this policy is *not* to make intermediaries pay for bailouts. In general, the revenue raised by  $\eta^*$  will not equal the size of the bailout package  $b^*$ , either in full or in expected value. Instead, the objective is to impose a marginal cost on each intermediary equal to the external effect its actions have in state  $\beta$ .

## 5.2 Illiquidity

How does the allocation vector  $\mathbf{c}^*$  compare with those derived under the previous policy regimes? One way of comparing the allocations is to look at the degree of illiquidity  $\rho$  as defined in (22).

**Proposition 7**  $\rho^N < \rho^* < \rho^B$  holds for all  $q > 0$ .

The Pigouvian approach reduces illiquidity in the financial system relative to the bailouts regime, but not as much as a no-bailouts policy does. It is straightforward to see why  $\rho^* < \rho^B$  holds: the introduction of the Pigouvian tax raises the cost of giving consumption to investors who withdraw before the state is revealed, which leads intermediaries to become more conservative in setting  $c_1$ . The comparison of  $\rho^*$  with  $\rho^N$  is more subtle. Equation (25) shows that the policy maker will choose  $\eta^*$  so that intermediaries equate the marginal value of private consumption before the state is revealed to the expected future marginal value of private consumption, exactly as they do under the no-bailouts policy in (19). However, these private consumption levels are different under the two regimes, especially in state  $\beta$ . Because the Pigouvian regime permits bailouts, the consumption levels  $(c_{1\beta}, c_{2\beta})$  are higher and, hence, the marginal value of this consumption,  $\mu_\beta$ , is lower than under a no-bailouts policy. As a result, intermediaries will be less conservative in setting  $c_1$  under the Pigouvian regime. In other words, the partial insurance offered by bailouts leads intermediaries to become more illiquid than under a no-bailouts policy, even when the incentive distortions associated with the bailouts are fully corrected by a Pigouvian tax.

### 5.3 Fragility and welfare

As with the previous regimes, the condition for financial fragility can be stated in terms of the elements of the best-response allocation  $\mathbf{c}^*$  associated with the particular profile of withdrawal strategies (1). This result is established in the following proposition, the proof of which follows those of Propositions 1 and 3 closely and is omitted.

**Proposition 8** *The financial system is fragile under the policy regime with bailouts and a Pigouvian tax on short-term liabilities if and only if  $c_1^* \geq c_{2\beta}^*$  holds.*

Let  $\Phi^*$  denote set of economies  $e$  that are fragile under the Pigouvian regime. The next proposition shows that this regime is unambiguously better for financial stability than the previous regimes.

**Proposition 9** *The set  $\Phi^*$  is strictly contained in both  $\Phi^B$  and  $\Phi^N$ .*

This result is illustrated in Figure 5, where  $\Phi^*$  corresponds to the darkest area at the top of panels (a) and (b). When the probability of a crisis is close to zero, the incentive distortion created by bailouts is small and the allocations  $\mathbf{c}^*$  and  $\mathbf{c}^B$  are nearly identical. In the limit as  $q$  goes to zero, therefore, the threshold value for  $\theta$  above which fragility obtains is the same for the sets  $\Phi^*$  and  $\Phi^B$ . As  $q$  increases, however, the policy maker now uses the fee  $\eta^*$  to correct the incentive distortion that arises under the bailouts regime. As a result, the threshold value for  $\theta$  no longer declines with  $q$  as in the set  $\Phi^B$ , but now increases slightly as the higher probability of a crisis raises the expected marginal value of consumption. For larger values of  $q$ , the combination of bailouts and a Pigouvian tax thus generates a substantially higher threshold for fragility than the policy regime with bailouts alone. In panel (c), where the public sector is relatively large, the financial system is never fragile under the Pigouvian regime.

The figure also shows the threshold value for  $\theta$  to be higher under the Pigouvian regime than under a no-bailouts policy for any value of  $q$ . To understand why, consider first the case where  $q$  is close to zero. In this case, the payments made to the first fraction  $\theta$  of investors to withdraw,  $c_1^*$  and  $c_1^N$ , are nearly identical under the two regimes, but the payments made to the remaining patient investors in the crisis state,  $c_{2\beta}^*$  and  $c_{2\beta}^N$ , are not. Under the Pigouvian regime, the bailout payments raise  $c_{2\beta}^*$ , which decreases the incentive for patient investors to run. When  $q$  is larger, the payments  $c_1^*$  and  $c_1^N$  begin to diverge as the policy maker sets the tax rate  $\tau$  differently in the two regimes, but the same basic effect is present. The magnitude of this effect depends on the size of the bailout payments which, in turn, depends on the amount of resources available to the policy maker. The

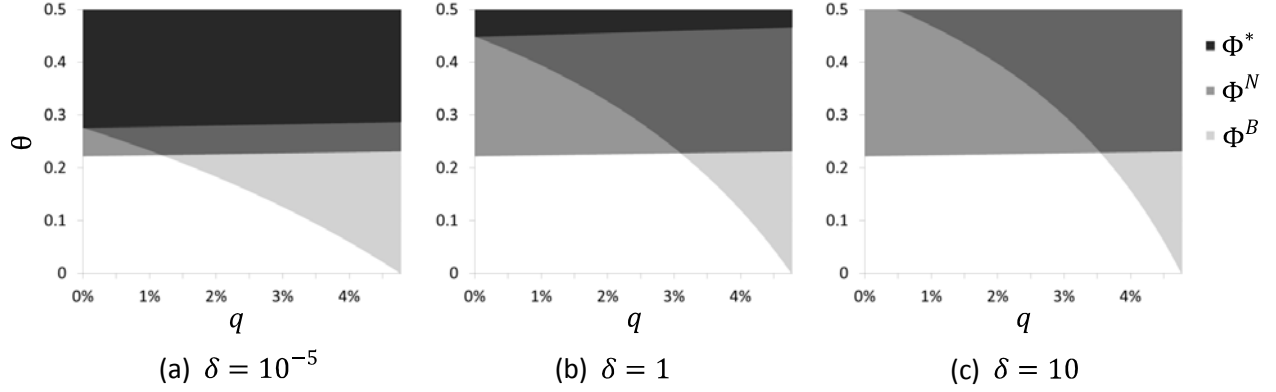


Figure 5:  $\Phi^*$  is strictly contained in both  $\Phi^B$  and  $\Phi^N$

gain in financial stability from adopting the Pigouvian regime instead of a no-bailouts policy is small when  $\delta$  is close to zero and the policy maker sets the tax rate  $\tau$  very low, but becomes much more significant for higher values of  $\delta$ .<sup>13</sup>

For an economy outside of the set  $\Phi^*$ , the optimal policy is clear: choosing the Pigouvian regime ensures that the financial system is not fragile and will generate the first-best allocation of resources in both states. For an economy in the set  $\Phi^*$ , Proposition 9 shows it will be fragile under all three policy regimes and, hence, the best regime is the one that generates the highest level of welfare  $W$  in the run equilibrium. The next proposition establishes that the Pigouvian regime is always best.

**Proposition 10** *For any  $e \in \Phi^*$  with  $q > 0$ ,  $W^* > \max \{W^B, W^N\}$ .*

## 5.4 Discussion

The Pigouvian approach raises welfare by combining the desirable features of each of the previous two regimes: it allows the socially-efficient risk sharing of the bailouts regime, but maintains proper incentives for intermediaries as in the no-bailouts regime. In fact, the proof of Proposition 10 given in the appendix shows that the Pigouvian regime generates the welfare-maximizing al-

<sup>13</sup> The set-theoretic approach to measuring fragility used here has a natural interpretation in terms of changes in the probability of a financial crisis. Suppose that at the beginning of period zero, the parameter values  $e$  are drawn at random from some probability distribution  $f$ . If the realized  $e$  is such that the economy is not fragile, investors do not run on the financial system in either state. If the economy is fragile, however, investors follow the strategy profile (1). The ex ante probability assigned to a crisis by this process will be strictly lower under the Pigouvian regime than under either of the other regimes for *any* probability distribution  $f$  that has full support. In this sense, Proposition 9 can be interpreted as saying that adopting the Pigouvian regime makes a financial crisis less likely to occur.

location of resources conditional on the withdrawal behavior specified in (1) and the restrictions imposed by sequential service. This fact implies that no alternative policy intervention can do better than the Pigouvian tax unless it eliminates the run equilibrium altogether. There may, however, be other ways of implementing this same outcome. Suppose, for example, the policy maker were able to place a cap on the payments intermediaries make to each of the first  $\theta$  investors who withdraw; this cap would be chosen at the time each withdrawal is made. It is straightforward to show that, in an equilibrium where investors follow (1), the policy maker would choose to set the cap at  $c_1^*$ . The full allocation resulting from intermediaries' and the policy maker's best responses would then be  $c^*$  and the fragile set would be  $\Phi^*$ , exactly as under the Pigouvian regime.<sup>14</sup>

Interestingly, this same outcome could be also achieved without regulation if the policy maker could commit to fixed bailout payments earlier in the timeline in Figure 1. Suppose that at the beginning of point (b) in the figure, after investors have made their withdrawal decisions but before any withdrawals take place, the policy maker could fix the complete profile  $\{b^j\}$  of amounts that will be paid in state  $\beta$ . To see why this change in timing matters, recall that the incentive distortion in this model arises because the policy maker would choose, ex post, to give larger bailout payments to intermediaries that are in worse financial condition. If the amounts  $b^j$  are fixed before intermediaries take any action, the bailout policy cannot possibly reward bad behavior and no incentive distortion arises. In this case, the remaining resources available to intermediary  $j$  in state  $\beta$  will no longer depend on aggregate conditions as in (8) but will instead equal

$$\psi_\beta^j = 1 - \tau - \theta c_1^j + b^j,$$

where the bailout payment  $b^j$  now enters as a lump-sum transfer that is independent of  $c_1^j$ . As a result, the value of  $\psi_\beta^j$  depends on bank  $j$ 's choice of  $c_1^j$ , which gives intermediary  $j$  an incentive to provision for state  $\beta$  even though it will receive a bailout. By setting the transfers  $\{b^j\}$  appropriately, the policy maker can provide the desired level of risk sharing between private and public consumption without distorting incentives. It is relatively straightforward to show that the equilibrium outcome in this case is again exactly the same as under the Pigouvian regime.

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<sup>14</sup> In other settings, especially where some characteristics of banks are unobserved by the policy maker, the outcomes under price-based and quantity-based regulation may be different. See Perotti and Suarez (2011) and Stein (2013) on this point.



This last result helps clarify the role of commitment in the analysis. While Propositions 5 and 6 show that committing to a no-bailouts policy is often undesirable, other forms of commitment would always be useful in this environment. If the bailout policy could be fixed at the very beginning of the timeline in Figure 1, before investors make their withdrawal decisions, the policy maker could eliminate the possibility of a run by committing to respond with fixed bailout payments that are sufficiently large. If the payments were instead chosen at the start of point (b), they would tend to be smaller and investors may run in equilibrium but, as described above, bailouts would not distort intermediaries' incentives. Note, however, that these policies would require committing to a plan that pre-specifies exactly how much each intermediary will receive in the event of a crisis. In practice, such commitment is likely difficult to achieve. The results in this section show how taxing intermediaries' short-term liabilities can partially substitute for commitment by removing distortions that arise when the policy maker cannot commit.

Implementing the Pigouvian regime raises a number of important issues that are beyond the scope of the present paper, such as how short-term liabilities should be measured and how the optimal tax rate can be determined. It is worth noting, however, that current policy efforts are addressing many of these issues. For example, the Basel III accords introduce a new framework for liquidity regulation that includes a methodology for measuring an intermediary's projected net cash outflows during a 30-day period of financial stress (see BCBS, 2013). This measure corresponds closely to the term  $\theta c_1^j$  in the model – the amount of funds that will flow out of the institution before the intermediary and policy maker are able to react – which is the appropriate base for the Pigouvian tax. Stein (2013) discusses how the regulatory tools created as part of this framework act as a tax on the activity of maturity transformation and thus have the same effect on incentives as the fee  $\eta$  used here. Kocherlakota (2010) suggests that the government could sell “rescue bonds” that pay off when one or more institutions is bailed out; the prices of these bonds could then be used to estimate the probability of a crisis, which is a key determinant of the optimal tax rate  $\eta^*$  in (27). The results in this section highlight the importance of these and related efforts to develop improved regulatory and prudential policy tools.

## 6 Concluding Remarks

The idea that bailouts can significantly distort the incentives of financial institutions and their investors has figured prominently in recent policy debates. One popular view holds that the best

way to deal with this problem is to restrict policy makers from engaging in future bailouts wherever possible. To evaluate this proposition, I have presented a model with three key features: (i) when there is aggregate risk in the economy, it is efficient for some of this risk to be borne by the public sector, (ii) the anticipation of being bailed out in the event of a crisis distorts the incentives of financial intermediaries, and (iii) investors are more likely to withdraw from the financial system when the potential loss they face in a crisis is larger. These features arise naturally in the environment I study here and seem likely to appear in a wide range of related settings.

It follows immediately from (i) that a strict no-bailouts policy cannot achieve an efficient allocation of resources in such a setting. If some risk should be borne by the public sector, achieving an efficient outcome requires allowing the policy maker to engage in bailouts if a crisis occurs. If bailouts are permitted, (ii) implies that policy makers should use prudential policy measures to offset the resulting distortion in incentives. In Section 5, I showed how placing a tax on intermediaries' short-term liabilities – and no restrictions on the bailout policy – is strictly better than adopting a no-bailouts policy: it generates higher welfare and a more stable financial system.

Proponents of the no-bailouts view may argue, however, that this result relies too heavily on the ability of policy makers and regulators to design and maintain an effective prudential policy regime. In practice, using taxes or other regulatory tools to precisely set incentives can be difficult. Intermediaries may, for example, be able to devise new types of liabilities or funding structures for which the appropriate regulatory treatment is not immediately clear. In a world where effectively controlling incentives through taxes/regulation is infeasible, one might be tempted to view a no-bailouts policy as at least representing a step in the right direction.

The primary message of the paper is that this view is often incorrect. The analysis highlights two important costs that limit the attractiveness of a no-bailouts policy. The first is a consequence of (i): by inefficiently concentrating all risk in the private sector, eliminating bailouts makes intermediaries too cautious from a social point of view and leads to an underprovision of financial services. Moreover, (iii) implies that removing the insurance provided by bailouts will tend to increase the susceptibility of the financial system to a crisis by making investors more prone to run at the first sign of trouble. In Section 4, I showed how adopting a no-bailouts policy can make investors strictly worse off by both lowering equilibrium welfare and making the financial system more fragile. In other words, the costs associated with a no-bailouts policy will, in many cases, outweigh the benefits.

## Appendix A. Proofs of Selected Propositions

**Proposition 1:** *The financial system is fragile under the bailouts regime if and only if (18) holds.*

*Proof:* The discussion in the text establishes that (18) is a sufficient condition for the financial system to be fragile, since it guarantees there exists an equilibrium in which investors follow strategy profile (1). What remains to be proven is that this condition is also necessary for fragility to arise.

To begin, note that any equilibrium strategy profile must satisfy two basic properties. First, all impatient investors must withdraw in period 1, since they receive no utility from consuming in period 2. Second, because (5) implies that intermediaries' best response to *any* strategy profile satisfies  $c_{1s} < c_{2s}$  in both states, all period-1 withdrawals that occur after intermediaries have observed the state must be made by impatient investors. For any strategy profile satisfying these two properties, let  $\varepsilon_s \in [0, 1 - \pi]$  denote the fraction of the first  $\theta$  withdrawals that are made by patient investors. (Note that (1) corresponds to the profile with  $\varepsilon_\alpha = 0$  and  $\varepsilon_\beta = 1 - \pi$ .) Definition 1 states that the financial system is fragile if there exists an equilibrium in which investors follow some strategy profile with  $\varepsilon_\beta > 0$ . I will show that whenever such an equilibrium exists, condition (18) holds.

Let  $\tilde{y}$  be any such strategy profile and let  $\tilde{c}$  denote the allocation generated by the best responses of intermediaries and the policy maker to this profile. This allocation is characterized by equations (4) - (6), (10), (13) - (15) and (17), where the fraction of remaining investors who are impatient after  $\theta$  withdrawals have been made,  $\hat{\pi}_s$ , is now given by

$$\hat{\pi}(\varepsilon_s) = \frac{\pi - \theta(1 - \varepsilon_s)}{1 - \theta} \quad \text{for } s = \alpha, \beta,$$

which is strictly increasing in the size of the run  $\varepsilon_s$ . If there is an equilibrium in which investors follow  $\tilde{y}$ , it must be the case that  $\tilde{c}$  satisfies

$$\tilde{c}_1 \geq \tilde{c}_{2\beta}, \tag{28}$$

that is, the patient investors who withdraw early in state  $\beta$  would not benefit by deviating and withdrawing in period 2. To establish that (18) is a necessary condition for fragility, therefore, it suffices to show that whenever (28) holds for some  $\tilde{y}$ , (18) also holds.

We can further limit the set of candidate strategy profiles  $\tilde{y}$  by noting that the restriction on

parameter values in (11) can be written as  $(1 - q) > 1/R$ , which clearly implies

$$(1 - q) \tilde{\mu}_\alpha > \frac{\tilde{\mu}_\alpha}{R}.$$

Combined with (5) and (10), this inequality implies  $u'(\tilde{c}_1) > u'(\tilde{c}_{2\alpha})$  and thus  $\tilde{c}_1 < \tilde{c}_{2\alpha}$ , which demonstrates that under the best-response allocation  $\tilde{c}$ , all patient investors strictly prefer to wait until period 2 to withdraw in state  $\alpha$ . We can, therefore, limit our attention to strategy profiles  $\tilde{y}$  with  $\varepsilon_\alpha = 0$ .

Since the function  $u$  exhibits constant relative risk aversion, expected utility preferences over pairs  $(c_{1s}, c_{2s})$  are homothetic and the solution to (3) satisfies the linear relationship

$$c_{2s} = R^{\frac{1}{\gamma}} c_{1s} \quad (29)$$

where  $\gamma > 1$  is the coefficient of relative risk aversion. Using the definitions of  $\psi_s$  in (13) - (14) together with (4), (15) and (29), we can write the resource constraint for each state  $s$  as

$$\theta c_1 + \left[ (1 - \theta) \left( \hat{\pi}(\varepsilon_s) R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}(\varepsilon_s) \right) \right] \frac{c_{2s}}{R} + g_s = 1. \quad (30)$$

Since  $R$  and  $\gamma$  are both greater than unity, the term in square brackets is strictly increasing in  $\varepsilon_s$ .

The remainder of the proof proceeds in three steps. First, I show that the shadow value of resources in state  $\alpha$  is lower under  $\tilde{c}$  than under  $c^B$ . I then show that the same is true in state  $\beta$ . Finally, I show how these two preliminary results combine to establish the proposition.

**Step 1:** Show  $\tilde{\mu}_\alpha \leq \mu_\alpha^B$ . This step of the proof is by contradiction. Suppose that  $\tilde{\mu}_\alpha > \mu_\alpha^B$  held. Then (5) and (10) would imply

$$\tilde{c}_1 < c_1^B \quad \text{and} \quad \tilde{c}_{2\alpha} < c_{2\alpha}^B. \quad (31)$$

Using the resource constraint (30) in state  $\alpha$  with  $\varepsilon_\alpha = 0$  for both strategy profiles, these inequalities would imply  $\tilde{g}_\alpha > g_\alpha^B$ . Equation (17) and the relationship  $\tau = g_\alpha$  would then imply

$$\tilde{\mu}_\alpha + \frac{q}{1-q} \tilde{\mu}_\beta \theta \frac{dc_1}{d\tau} < \mu_\alpha^B + \frac{q}{1-q} \mu_\beta^B \theta \frac{dc_1}{d\tau}.$$

It is straightforward to show from (10) that the derivative  $dc_1/d\tau$  is negative and the same under both allocations. Since  $\tilde{\mu}_\alpha > \mu_\alpha^B$  is assumed to hold, this expression would imply  $\tilde{\mu}_\beta > \mu_\beta^B$ .

However, (5), (6) and the relationship  $g_\beta = \tau - b$  would then imply

$$\tilde{c}_{2\beta} < c_{2\beta}^B \quad \text{and} \quad \tilde{g}_\beta < g_\beta^B.$$

Using  $\varepsilon_\beta \leq 1 - \pi$ , we would then have

$$\begin{aligned} & (1 - \theta) \left( \hat{\pi}(\varepsilon_\beta) R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}(\varepsilon_\beta) \right) \frac{\tilde{c}_{2\beta}}{R} + \tilde{g}_\beta \\ & < (1 - \theta) \left( \hat{\pi}(1 - \pi) R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}(1 - \pi) \right) \frac{c_{2\beta}^B}{R} + g_\beta^B, \end{aligned}$$

which, using the resource constraint (30) in state  $\beta$ , would yield  $\tilde{c}_1 > c_1^B$ , contradicting (31) above.

Therefore,  $\tilde{\mu}_\alpha \leq \mu_\alpha^B$  must hold.

**Step 2:** Show  $\tilde{\mu}_\beta \leq \mu_\beta^B$ . The result from step 1, together with (5) and (10), implies both  $\tilde{c}_1 \geq c_1^B$  and  $\tilde{c}_{2\alpha} \geq c_{2\alpha}^B$ . The resource constraint (30) in state  $\alpha$  then implies

$$\tilde{g}_\alpha \leq g_\alpha^B, \tag{32}$$

which, combined with (17), yields

$$\tilde{\mu}_\alpha + \frac{q}{1-q} \tilde{\mu}_\beta \theta \frac{dc_1}{d\tau} \geq \mu_\alpha^B + \frac{q}{1-q} \mu_\beta^B \theta \frac{dc_1}{d\tau}.$$

Since  $\tilde{\mu}_\alpha \leq \mu_\alpha^B$  holds and the derivative  $dc_1/d\tau$  is negative (and the same under both allocations), this expression implies  $\tilde{\mu}_\beta \leq \mu_\beta^B$ , as desired.

**Step 3:** Show that  $\tilde{c}_1 \geq \tilde{c}_{2\beta}$  implies (18). The previous step shows how  $\tilde{\mu}_\alpha \leq \mu_\alpha^B$  implies (32) and thus

$$v'(\tilde{g}_\alpha) \geq v'(g_\alpha^B).$$

Also, using (6), the result from step 2 that  $\tilde{\mu}_\beta \leq \mu_\beta^B$  holds implies that we have  $\tilde{g}_\beta \geq g_\beta^B$  and thus

$$v'(\tilde{g}_\beta) \leq v'(g_\beta^B).$$

Combining these two inequalities yields

$$\frac{v'(\tilde{g}_\alpha)}{v'(\tilde{g}_\beta)} \geq \frac{v'(g_\alpha^B)}{v'(g_\beta^B)}.$$

Using (6) and (17), we then have

$$\frac{\tilde{\mu}_\alpha + \frac{q}{1-q}\tilde{\mu}_\beta\theta\frac{dc_1}{d\tau}}{\tilde{\mu}_\beta} \geq \frac{\mu_\alpha^B + \frac{q}{1-q}\mu_\beta^B\theta\frac{dc_1}{d\tau}}{\mu_\beta^B}$$

or, again using the fact that the derivative  $dc_1/d\tau$  is the same under both allocations,

$$\frac{\tilde{\mu}_\alpha}{\tilde{\mu}_\beta} \geq \frac{\mu_\alpha^B}{\mu_\beta^B}.$$

Together with (5) and (10), this inequality implies

$$\frac{u'(\tilde{c}_1)}{u'(\tilde{c}_{2\beta})} \geq \frac{u'(c_1^B)}{u'(c_{2\beta}^B)}.$$

Using the homotheticity of preferences, we then have

$$\frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \leq \frac{c_1^B}{c_{2\beta}^B}.$$

It follows immediately from this inequality that (28) implies (18), as desired. ■

**Proposition 2:** For any  $e \in \Phi^B$ , we have  $(c_{1\beta}^B, c_{2\beta}^B, g_\beta^B) \ll (c_{1\alpha}^B, c_{2\alpha}^B, g_\alpha^B)$ .

*Proof:* Consider any economy in  $\Phi^B$ . Condition (11) can be written as

$$(1-q)\mu_\alpha^B > \frac{1}{R}\mu_\alpha^B,$$

which, together with (5) and (10), implies  $u'(c_1^B) > u'(c_{2\alpha}^B)$ . Combining this inequality with (18) yields  $c_{2\beta}^B \leq c_1^B < c_{2\alpha}^B$ . The first-order conditions in (5) then imply

$$\mu_\beta^B > \mu_\alpha^B \quad \text{and} \quad c_{1\beta}^B < c_{1\alpha}^B, \tag{33}$$

which establishes that both elements of private consumption are smaller in state  $\beta$  than in state  $\alpha$ .

To show that public consumption is also lower in state  $\beta$ , note that (17), together with the relationship  $g_\alpha = \tau$  and the fact that the derivative  $dc_1/d\tau$  is negative, implies

$$v'(g_\alpha^B) < \mu_\alpha^B.$$

Combining this result with the first inequality in (33) and equation (6) yields

$$v'(g_\alpha^B) < \mu_\beta^B = v'(g_\beta^B),$$

which immediately implies  $g_\beta^B < g_\alpha^B$  and concludes the proof. ■

**Proposition 4:**  $\rho^N < \rho^B$  holds for all  $q > 0$ .

*Proof:* This result follows from Proposition 7, which is proved below. ■

**Proposition 6:** For any  $(R, \pi, \theta, u, v)$ , there exists  $\bar{q} > 0$  such that  $q < \bar{q}$  and  $e \in \Phi^B$  implies both  $e \in \Phi^N$  and  $W^N < W^B$ .

*Proof:* First, note that using the definitions of  $\psi_s$  in (13) - (14) together with (4), (15) and (29), we can write the resource constraint for each state  $s$  as

$$\theta c_1 + \left[ (1 - \theta) \left( \hat{\pi}_s R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_s \right) \right] \frac{c_{2s}}{R} + g_s = 1. \quad (34)$$

For any  $(R, \pi, \theta, u, v)$ , we can consider the best-response allocations  $\mathbf{c}^B$  and  $\mathbf{c}^N$  as functions of the probability  $q$ . Note that, in the limit as  $q$  goes to zero, conditions (10) and (19) become equivalent, as do conditions (17) and (21). Combined with (5) and (34) for  $s = \alpha$ , the convergence of these two pairs of conditions implies

$$\lim_{q \rightarrow 0} (c_1^N(q), c_{1\alpha}^N(q), c_{2\alpha}^N(q), g_\alpha^N(q)) = \lim_{q \rightarrow 0} (c_1^B(q), c_{1\alpha}^B(q), c_{2\alpha}^B(q), g_\alpha^B(q)). \quad (35)$$

Intuitively, as the probability of a crisis goes to zero, the effect of events in the crisis state on the allocation of resources in normal times disappears.

The allocation of resources in state  $\beta$  remains different across regimes, however. In particular, public consumption in state  $\beta$  is lower under the bailouts regime because some public funds are transferred to the intermediaries. This fact can be seen by combining (35) with the result  $g_\beta^B < g_\alpha^B$  from Proposition 2 and the no-bailouts condition (20) to obtain

$$\lim_{q \rightarrow 0} g_\beta^N(q) > \lim_{q \rightarrow 0} g_\beta^B(q). \quad (36)$$

Looking at the resource constraint (34) for  $s = \beta$ , equations (35), (36) and the definitions of  $\hat{\pi}_s$  in (2) imply that in the limit as  $q$  goes to zero,  $c_{2\beta}$  must be lower in the no-bailouts regime,

$$\lim_{q \rightarrow 0} c_{2\beta}^N(q) < \lim_{q \rightarrow 0} c_{2\beta}^B(q).$$

Together, this inequality and (35) imply that there exists some  $\bar{q} > 0$  such that

$$\frac{c_1^N(q)}{c_{2\beta}^N(q)} > \frac{c_1^B(q)}{c_{2\beta}^B(q)} \quad \text{for all } q < \bar{q}.$$

For any  $q < \bar{q}$ , if  $e \in \Phi^B$  then  $c_1^B(q) \geq c_{2\beta}^B(q)$  holds by definition. The inequality above then implies  $c_1^N(q) > c_{2\beta}^N(q)$  and, hence,  $e \in \Phi^N$  also holds, establishing the first part of the proposition.

For the second part of the proposition, note that (35) implies the welfare levels associated with the run equilibrium allocations under the two regimes converge as  $q$  goes to zero,

$$\lim_{q \rightarrow 0} W^N(q) = \lim_{q \rightarrow 0} W^B(q).$$

As  $q$  approaches zero, both the incentive distortions and the welfare loss associated with a potential crisis disappear. The proposition will be established, therefore, if we can show that welfare initially falls faster under the no-bailouts regime as  $q$  rises, that is, if we can show

$$\lim_{q \rightarrow 0} \frac{dW^N(q)}{dq} < \lim_{q \rightarrow 0} \frac{dW^B(q)}{dq}. \quad (37)$$

The derivative for the bailouts regime can be written as

$$\lim_{q \rightarrow 0} \frac{dW^B(q)}{dq} = -V_\alpha (1 - g_\alpha^B - \theta c_1^B) - v(g_\alpha^B) + V_\beta (1 - g_\beta^B - \theta c_1^B) + v(g_\beta^B).$$

Using (35), we can write the corresponding derivative under a no-bailouts policy as

$$\lim_{q \rightarrow 0} \frac{dW^N(q)}{dq} = -V_\alpha (1 - g_\alpha^B - \theta c_1^B) - v(g_\alpha^B) + V_\beta (1 - g_\alpha^B - \theta c_1^B) + v(g_\alpha^B).$$

Notice that the first two terms in these derivatives, which are based on the allocation of resources in state  $\alpha$ , are the same. The last two terms differ, however, because under a no-bailouts policy the level of the public good must be equal across states:  $g_\beta^N = g_\alpha^N$ . Both of these values converge to  $g_\alpha^B$  as  $q$  goes to zero, as indicated in the expression above.

The final step is to note that the bailout policy characterized by (6) sets the level of public consumption in state  $\beta$  to maximize the sum of these last two terms. Since Proposition 2 establishes that  $g_\alpha^B \neq g_\beta^B$ , we must, therefore, have

$$V_\beta (1 - g_\beta^B - \theta c_1^B) + v(g_\beta^B) > V_\beta (1 - g_\alpha^B - \theta c_1^B) + v(g_\alpha^B)$$

and, hence, the inequality in (37) holds, as desired. ■



**Proposition 7:**  $\rho^N < \rho^* < \rho^B$  holds for all  $q > 0$ .

*Proof:* We first establish  $\rho^* < \rho^B$ . Since the multipliers  $\mu_\alpha^*$  and  $\mu_\beta^*$  are always strictly positive, we clearly have

$$(1 - q) + q \frac{\mu_\beta^*}{\mu_\alpha^*} > (1 - q),$$

which can be rewritten as

$$\frac{(1 - q) \mu_\alpha^* + q \mu_\beta^*}{\mu_\alpha^*} > \frac{(1 - q) \mu_\alpha^B}{\mu_\alpha^B}.$$

Using (5), (10) and (19), this inequality implies

$$\frac{u'(c_1^*)}{u'(c_{2\alpha}^*)} > \frac{u'(c_1^B)}{u'(c_{2\alpha}^B)}.$$

Using the homotheticity of preferences over pairs  $(c_1, c_{2\alpha})$ , we then have

$$\frac{c_1^*}{c_{2\alpha}^*} < \frac{c_1^B}{c_{2\alpha}^B}. \quad (38)$$

The resource constraint (34) for state  $\alpha$  can be written as

$$\rho^{-1} = \frac{1 - g_\alpha}{\theta c_1} = 1 + \frac{(1 - \theta)}{\theta} \left( \hat{\pi}_\alpha R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\alpha \right) \frac{1}{R} \left( \frac{c_{2\alpha}}{c_1} \right). \quad (39)$$

Using this equation, the inequality in (38) immediately implies  $\rho^* < \rho^B$ , as desired.

Next, we establish the first inequality in the proposition,  $\rho^N < \rho^*$ . An argument similar to that in the proof of Proposition 2 can be used to show  $g_\beta^* < g_\alpha^*$  holds. Using the resource constraint (34) for each state, this inequality implies

$$\left( \hat{\pi}_\beta R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\beta \right) \frac{c_{2\beta}^*}{R} > \left( \hat{\pi}_\alpha R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\alpha \right) \frac{c_{2\alpha}^*}{R}.$$

Under a no-bailout policy,  $g_\beta^N = g_\alpha^N$  holds by definition and the resource constraints imply

$$\left( \hat{\pi}_\beta R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\beta \right) \frac{c_{2\beta}^N}{R} = \left( \hat{\pi}_\alpha R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\alpha \right) \frac{c_{2\alpha}^N}{R}.$$

Combining these two relationships yields

$$\frac{c_{2\beta}^*}{c_{2\alpha}^*} > \frac{c_{2\beta}^N}{c_{2\alpha}^N},$$

which, by the homotheticity of preferences, implies

$$\frac{u'(c_{2\beta}^*)}{u'(c_{2\alpha}^*)} < \frac{u'(c_{2\beta}^N)}{u'(c_{2\alpha}^N)} \quad \text{or} \quad \frac{\mu_\beta^*}{\mu_\alpha^*} < \frac{\mu_\beta^N}{\mu_\alpha^N}. \quad (40)$$

From the latter inequality, we clearly have

$$(1 - q) + q \frac{\mu_\beta^*}{\mu_\alpha^*} < (1 - q) + q \frac{\mu_\beta^N}{\mu_\alpha^N},$$

which can be rewritten as

$$\frac{(1 - q)\mu_\alpha^* + q\mu_\beta^*}{R^{-1}\mu_\alpha^*} < \frac{(1 - q)\mu_\alpha^N + q\mu_\beta^N}{R^{-1}\mu_\alpha^N}.$$

Using (5), (10) and (19), we then have

$$\frac{u'(c_1^*)}{u'(c_{2\alpha}^*)} < \frac{u'(c_1^N)}{u'(c_{2\alpha}^N)}.$$

Again using the homotheticity of preferences, this last inequality implies

$$\frac{c_1^*}{c_{2\alpha}^*} > \frac{c_1^N}{c_{2\alpha}^N},$$

which, using (39), yields  $\rho^* > \rho^N$ , as desired. ■

**Proposition 9:** *The set  $\Phi^*$  is strictly contained in both  $\Phi^B$  and  $\Phi^N$ .*

*Proof:* The proof is divided into four steps.

**Step 1:** Derive bounds for the derivative  $dc_1^B/d\tau$ . Differentiating the resource constraint

$$\theta c_1^B + (1 - \theta) \left( \hat{\pi}_\alpha R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\alpha \right) \frac{c_{2\alpha}^B}{R} = 1 - \tau$$

with respect to  $\tau$  yields

$$\theta \frac{dc_1^B}{d\tau} + (1 - \theta) \left( \hat{\pi}_\alpha R^{\frac{\gamma-1}{\gamma}} + 1 - \hat{\pi}_\alpha \right) \frac{1}{R} \frac{dc_{2\alpha}^B}{d\tau} = -1. \quad (41)$$

Next, combining (5) and (10), then differentiating with respect to  $\tau$  yields

$$\frac{dc_{2\alpha}^B}{d\tau} = \frac{1}{1 - q} \frac{u''(c_1^B)}{Ru''(c_{2\alpha}^B)} \frac{dc_1^B}{d\tau}. \quad (42)$$

Using (5), (10) and that fact that  $u$  is of the constant relative risk aversion form, it is straightforward

to show

$$\frac{u''(c_1^B)}{Ru''(c_{2\alpha}^B)} = (1-q)^{\frac{\gamma+1}{\gamma}} R^{\frac{1}{\gamma}}.$$

Substituting this relationship and (42) into (41) and solving yields

$$\frac{dc_1^B}{d\tau} = -\frac{1}{\theta + \chi}$$

where

$$\chi \equiv (1-q)^{\frac{1}{\gamma}} (1-\theta) \left( \hat{\pi}_\alpha + (1-\hat{\pi}_\alpha) R^{\frac{1-\gamma}{\gamma}} \right) > 0.$$

Note that  $\chi > 0$  implies the following bounds

$$-1 < \theta \frac{dc_1^B}{d\tau} < 0.$$

Combined with (17), the first of these inequalities implies that for any  $q > 0$ , we have

$$v'(g_\alpha^B) > \mu_\alpha^B - \frac{q}{1-q} \mu_\beta^B. \quad (43)$$

**Step 2:** Show that  $c_1^B > c_1^*$  holds for all  $q > 0$ . Suppose, to the contrary, that  $c_1^B \leq c_1^*$  held for some  $q > 0$ . Then (10) and (25) would imply

$$(1-q) \mu_\alpha^B \geq (1-q) \mu_\alpha^* + q \mu_\beta^*. \quad (44)$$

In addition,  $1 - \theta c_1^B \geq 1 - \theta c_1^*$  would imply  $\mu_\beta^B < \mu_\beta^*$ . Combining these two inequalities yields

$$\mu_\alpha^B - \frac{q}{1-q} \mu_\beta^B > \mu_\alpha^*.$$

The inequality in (43) and condition (26) would then imply  $v'(g_\alpha^B) > v'(g_\alpha^*)$  and, hence,

$$g_\alpha^B < g_\alpha^*. \quad (45)$$

Note that (44) would also immediately imply  $\mu_\alpha^B > \mu_\alpha^*$  which, using (5), would imply

$$c_{2\alpha}^B < c_{2\alpha}^*. \quad (46)$$

However, combining the assumed relationship  $c_1^B \leq c_1^*$  with (45) and (46), we would have

$$\theta c_1^B + (1-\theta) \left( \frac{\hat{\pi}_\alpha}{R^{\frac{1-\gamma}{\gamma}}} + 1 - \hat{\pi}_\alpha \right) \frac{c_{2\alpha}^B}{R} + g_\alpha^B < \theta c_1^* + (1-\theta) \left( \frac{\hat{\pi}_\alpha}{R^{\frac{1-\gamma}{\gamma}}} + 1 - \hat{\pi}_\alpha \right) \frac{c_{2\alpha}^*}{R} + g_\alpha^*.$$

In other words, all components of the resource constraint in state  $\alpha$  would be smaller under the bailouts regime than under the Pigouvian regime, which contradicts the fact that the resource constraint holds with equality in both cases. Therefore,  $c_1^B > c_1^*$  must hold.

**Step 3:** Show  $\Phi^B \supset \Phi^*$ . Consider any economy in  $\Phi^*$ , that is, any economy for which  $c_1^* \geq c_{2\beta}^*$  holds. The result from Step 2 tells us that  $c_1^B > c_{2\beta}^*$  also holds for this economy, which, together with (5) and (10), implies

$$(1 - q) \mu_\alpha^B < \frac{1}{R} \mu_\beta^*.$$

Furthermore, it follows from Step 2 that  $1 - \theta c_1^B < 1 - \theta c_1^*$  holds, which implies  $\mu_\beta^B > \mu_\beta^*$ . Therefore, we have

$$(1 - q) \mu_\alpha^B < \frac{1}{R} \mu_\beta^B, \quad (47)$$

which, using (5) and (10), implies  $c_1^B > c_{2\beta}^B$  and, hence, the economy is also in  $\Phi^B$ . Moreover, the fact that the inequality in (47) is strict implies that the inclusion relationship is also strict: there exist economies for which  $c_1^*$  is slightly smaller than  $c_{2\beta}^*$ , but (47) still holds. Alternatively, it is easy to find examples of economies that belong to  $\Phi^B$  but not to  $\Phi^*$ ; see Figure 5.

**Step 4:** Show  $\Phi^N \supset \Phi^*$ . For any  $e \in \Phi^*$ , we know that  $c_1^* \geq c_{2\beta}^*$  holds. Using (5) and (25), this condition can be rewritten as

$$\frac{\mu_\alpha^*}{\mu_\beta^*} \leq \frac{R^{-1} - q}{1 - q}.$$

The second inequality in (40) then implies

$$\frac{\mu_\alpha^N}{\mu_\beta^N} < \frac{R^{-1} - q}{1 - q}.$$

Straightforward algebra, using (5) and (19), shows that this latter inequality is equivalent to  $c_1^N > c_{2\beta}^N$ , meaning that  $e \in \Phi^N$  also holds. Once again, the fact that this inequality is strict implies that the inclusion relationship is also strict: there exist economies for which  $c_1^*$  is slightly smaller than  $c_{2\beta}^*$ , but  $c_1^N \geq c_{2\beta}^N$  still holds. Alternatively, it is easy to find examples of economies that belong to  $\Phi^N$  but not to  $\Phi^*$ ; see Figure 5. Together, the results in Steps 3 and 4 establish the proposition. ■

**Proposition 10:** For any  $e \in \Phi^*$  with  $q > 0$ ,  $W^* > \max \{W^B, W^N\}$ .

*Proof:* I will show that the equilibrium allocation of resources under the Pigouvian regime attains the highest possible level of welfare conditional on investors following strategy profile (1). It then follows that the welfare levels associated with  $c^B$  and  $c^N$  are both strictly lower.

Suppose a benevolent planner could control all endowments and operate both the intermediation technology and the public sector. This planner cannot control investors' withdrawal decisions and is subject to the same informational constraints as are intermediaries and the policy maker. In particular, the planner observes the state  $s$  only after a fraction  $\theta$  of investors have withdrawn. If investors follow the partial-run strategy profile in (1), what is the planner's best response?

Because investors are risk averse, the planner will choose to give the same amount of consumption,  $\widehat{c}_1$ , to each investor who withdraws before the state is revealed. Once she has observed the state to be  $s$ , the planner will choose to give common amounts  $(\widehat{c}_{1s}, \widehat{c}_{2s})$  to each of the remaining impatient and patient investors, respectively, and will provide an amount  $g_s$  of the public good. In other words, the planner's best response to the strategy profile (1) can be summarized by a vector

$$\mathbf{c} \equiv \left\{ c_1, \{c_{1s}, c_{2s}, g_s\}_{s=\alpha, \beta} \right\}$$

in exactly the same way that the best responses of intermediaries and the policy maker can in the decentralized economy. The elements of this vector will be chosen to maximize

$$\begin{aligned} & \theta u(c_1) + (1 - q) \left( (1 - \theta) [\widehat{\pi}_\alpha u(c_{1\alpha}) + (1 - \widehat{\pi}_\alpha) u(c_{2\alpha})] + v(g_\alpha) \right) + \\ & q \left( (1 - \theta) [\widehat{\pi}_\beta u(c_{1\beta}) + (1 - \widehat{\pi}_\beta) u(c_{2\beta})] + v(g_\beta) \right) \end{aligned}$$

subject to the resource constraints

$$\theta c_1 + (1 - \theta) \left( \widehat{\pi}_s c_{1s} + (1 - \widehat{\pi}_s) \frac{c_{2s}}{R} \right) + g_s \leq 1 \quad (48)$$

for  $s = \alpha, \beta$ . Letting  $\mu_s$  denote the multiplier on the resource constraint in state  $s$ , the first-order conditions characterizing the solution to this problem are

$$u'(c_1) = (1 - q) \mu_\alpha + q \mu_\beta \quad \text{and} \quad (49)$$

$$u'(c_{1s}) = R u'(c_{2s}) = v'(g_s) = \mu_s \quad (50)$$

for  $s = \alpha, \beta$ . It is straightforward to show that the equations characterizing the equilibrium allocation vector  $\mathbf{c}^*$  under the Pigouvian regime, which are (4) - (6), (13) - (15), (25) and (26), can be reduced to precisely equations (48) - (50). This fact demonstrates that  $\mathbf{c}^*$  solves the problem of the benevolent planner and, since this solution is unique, must generate strictly higher welfare than both  $\mathbf{c}^B$  and  $\mathbf{c}^N$ . ■

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