Problem Set #1

Economic Growth
Spring 2005

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1) Consider the optimal growth problem from class, with $u(c) = \ln(c)$ and $n = 0$, and compare the following two situations. In the baseline case, the island has $k_0 < k^*$ trees when Crusoe arrives. In the modified case, the island only has half as many trees $\left(\frac{1}{2}k_0\right)$ when Crusoe arrives. Everything else is the same in the two cases.

a) Draw the time paths of k and c for these two cases. How do they compare?

b) Suppose that we look at the growth rate of consumption (that is, $\dot{c}/c$) in each of these two cases. At time $t = 0$, in which case will the growth rate be higher? Why? (Give some intuition for your answer).

2) Now consider the following changes in the model. First, instead of the general harvesting function $Y(t) = F(K(t), N(t))$, use the Cobb-Douglas function $Y(t) = K(t)^{\alpha} N(t)^{1-\alpha}$. Second, instead of the utility function $u(c) = \ln(c)$, the Crusoe household's preferences are represented by

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta},$$

with $\theta > 0$. Note that when $\theta = 1$, this function is not well defined (it gives "zero divided by zero" for every value of c). However, as $\theta \to 1$, the function converges to $\ln(c)$. Finally, assume $n > 0$.

a) Write down the optimal growth problem, the Hamiltonian function for this problem, and the 3 first-order conditions. Also write down the transversality condition.

b) Solve the first-order conditions to get a system of two differential equations in the variables (k, c). How do these differ from the equations we saw in class?

c) Do the following comparative dynamics exercise: $n' > n$. Proceed exactly as we did in class. Draw the phase diagram for the baseline case, and suppose that k_0 is equal to k^* for this case. Then draw the modified phase diagram, indicating what has changed with the higher value of n. Draw the modified time paths of k and c, indicating how they compare with the baseline time paths. What effect does the increase in n have? Why? (Give an intuitive answer.)