Solutions to Problem Set #4

Economic Growth — Professor Todd Keister
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Consider the Ramsey model with productivity growth. Suppose the government imposes a sales tax on firms: the representative firm must pay a fraction τ of its output to the government. Assume there is population growth $(n > 0)$. Find the competitive equilibrium of this economy, using the following steps.

a) Write down the 4 equations the characterize the optimal behavior of the representative household.

Because this tax policy has no direct effect on the household, the equations are the usual:

$$\begin{aligned} \dot{c}(t) &= \frac{1}{\theta}\left[r(t) - \rho\right] c(t) && (1) \\ \dot{a}(t) &= w(t) + r(t)\,a(t) - c(t) - na(t) && (2) \\ \lim_{t\to\infty} \mu(t)\,a(t) &= 0 && (3) \\ c(t) &\geq -B \text{ for all } t && (4) \end{aligned}$$

b) Write down firm's maximization problem and the first-order conditions for this problem. Translate these conditions into intensive form.

The firm's problem is

$$\max\ (1-\tau)\,F(K(t), A(t)\,L(t)) - w(t)\,L(t) - R(t)\,K(t)$$

subject to

$$K(t), L(t) \geq 0$$

Taking the first-order conditions and translating them into intensive form in the usual way yields

$$\begin{aligned} (1-\tau)\,F_K(k(t), A(t)) &= R(t) && (5) \\ (1-\tau)\left[F(k(t), A(t)) - k(t)\,F_K(k(t), A(t))\right] &= w(t) && (6) \end{aligned}$$

c) What are the equilibrium conditions for this economy?

The tax has no direct effect on banks nor on the labor-market clearing condition:

$$\begin{aligned} N(t) &= L(t) && (7) \\ a(t) &= k(t) && (8) \\ R(t) &= r(t) + \delta && (9) \end{aligned}$$

d) Let $\phi(t)$ denote the per-capita revenue of the government at time t. What is the government's budget constraint?

In total (extensive) form, we have

$$\phi(t) N(t) = \tau F(K(t), A(t) L(t)).$$

Using (7) and dividing both sides by $L(t)$, we convert this equation into intensive form

$$\phi(t) = \tau F(k(t), A(t)). \tag{10}$$

e) Combine your answers to parts (a) - (d) to get differential equations for the variables k and c.

Doing the usual substitutions and simplifications, we arrive at

$$\begin{aligned} \dot{c}(t) &= \frac{1}{\theta}\left[(1-\tau) F_K(k(t), A(t)) - \delta - \rho\right] c(t) \\ \dot{k}(t) &= (1-\tau) F(k(t), A(t)) - c(t) - (\delta + n) k(t) \end{aligned}$$

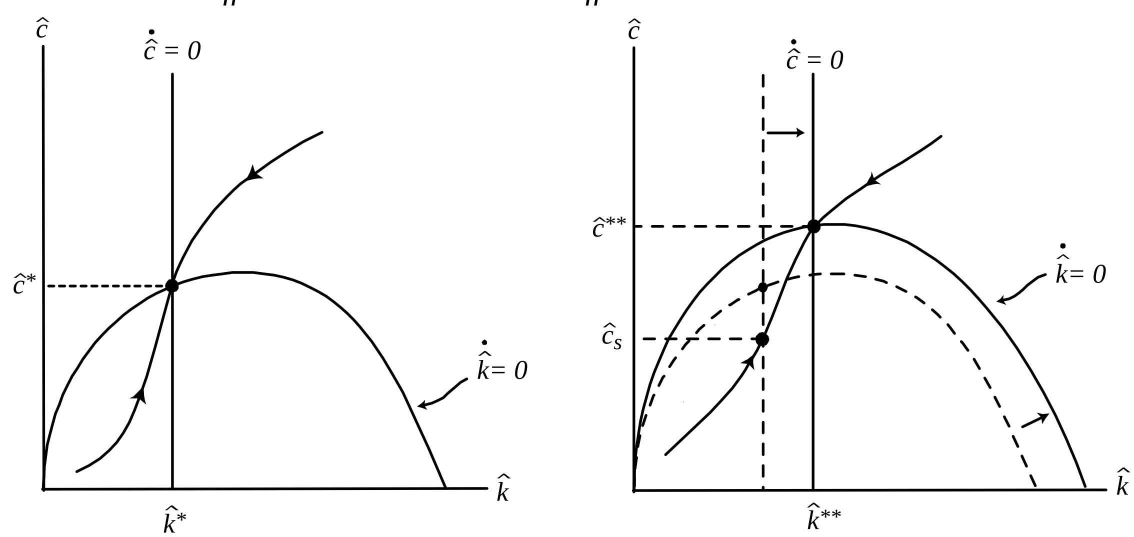
Notice what the second equation says: the government it taking away a fraction τ of output (per worker). This is consistent with the budget constraint we wrote for the government in part (d).

f) Translate these two equations into per-effective-workers terms (so that we have differential equations for the variables $\widehat{k}$ and $\widehat{c}$).

Performing the usual steps here leads to

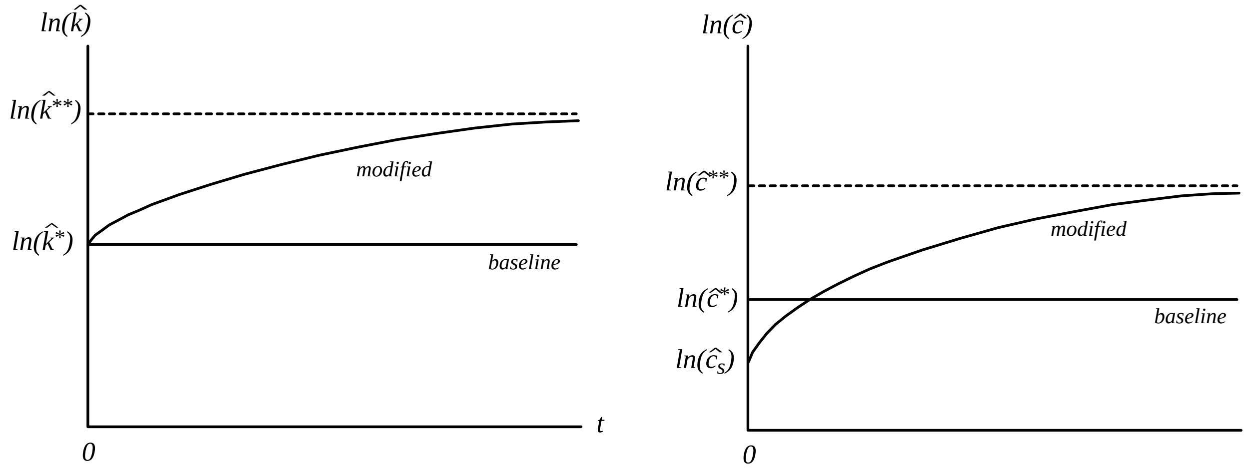
$$\begin{aligned} \dot{\widehat{c}}(t) &= \frac{1}{\theta}\left[(1-\tau) f'\left(\widehat{k}(t)\right) - \delta - \rho - \theta g\right] \widehat{c}(t) \\ \dot{\widehat{k}}(t) &= (1-\tau) f\left(\widehat{k}(t)\right) - \widehat{c}(t) - (\delta + n + g) \widehat{k}(t) \end{aligned}$$

g) Do the following comparative dynamics exercise: $\tau' < \tau$. Draw (i) the phase diagrams, (ii) the time paths of $\widehat{k}$ and $\widehat{c}$, and (iii) the time paths of k and c for both cases. If necessary, assume that the substitution effect dominates the income effect.

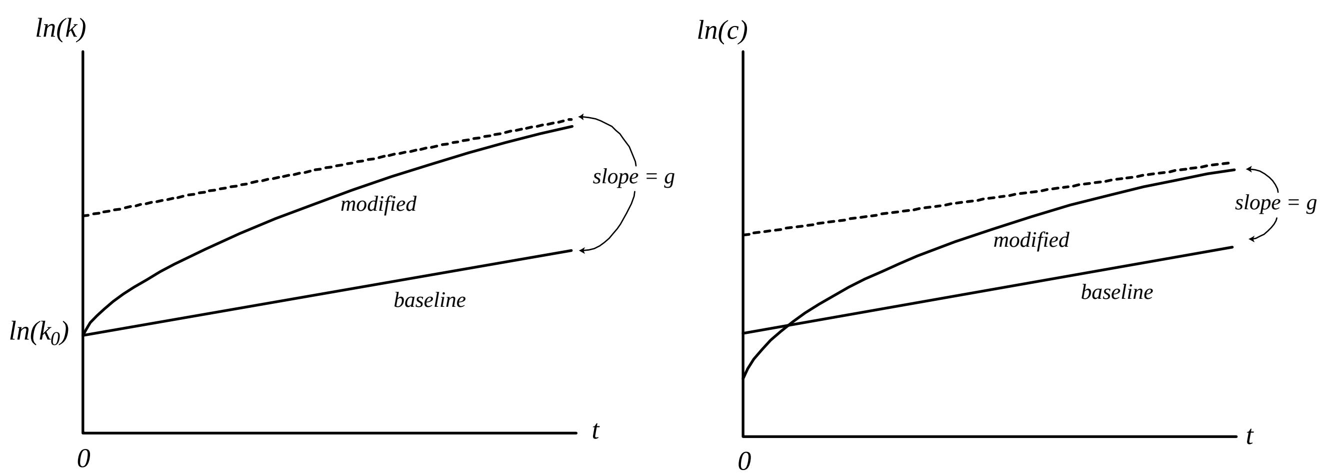


The lower tax rate in the modified case causes the isocline for c to shift to the right and the isocline for k to rotate upwards, as shown in the phase diagram above.

The lower tax rate makes investment more productive and therefore makes consuming in the future relatively less expensive. The substitution effect then points toward consuming less today (and more in the future). The lower tax rate also makes households richer, so that the income effect points toward consuming more today (and more in the future). If the substitution effect dominates, we must have $c_S < c^*$. The time paths of $\widehat{k}$ and $\widehat{c}$ are:



Finally, the time paths of k and c are:



4) Consider the following data for Mexico in 1965 and 1990:

Year	Population	Real GDP	Labor Force	Capital Stock
1965	44,854,000	150,305,754,000	13,029,278	25,186,454,808
1990	81,724,000	476,205,748,000	27,992,344	81,969,979,536

a) What is the average annual growth rate of real GDP over this period? Do the growth accounting analysis using this data, and using $\alpha = 0.69$. [The data is drawn from the Penn World Table, ver. 5.6. The Real GDP and Capital Stock variables are measured in 1985 US$. The Labor Force variable is an estimate of the number of full-time equivalent workers employed during the year. The Capital Stock variable is an estimate of the value of all producer durables.]

Using the data above and the growth-rate formula from class, we have

1965-1990	GDP	Capital	Labor	TFP
Growth rate (calculated from data)	0.0461	0.0472	0.0306	
Contribution (mulitply by α or $(1-\alpha)$) and calculate TFP growth as a residual)		0.0326	0.0095	0.0041
Fraction of total contribution		(70.6%)	(20.6%)	(8.8%)

Notice that the data on the size of the population is irrelevant for this exercise.

b) In class, we saw the following information for the period 1940-1980 (taken from p. 381 of Barro & Sala-i-Martin):

1940-1980 :	GDP	Capital	Labor	TFP
Growth rate	0.0630	0.0370	0.0468	
Contribution		0.0255	0.0145	0.0230
Fraction of total contribution		(40.5%)	(23.0%)	(36.5%)

How do the results for the two time periods compare? In particular, to what factor does the analyis attribute most of the slowdown of GDP growth in Mexico?

For the more recent period, the growth rate of GDP is lower by 0.0158, that is, one-and-a-half percentage points. By comparing the two growth-accounting tables, we can see what factors contributed to this decrease in the growth rate. The growth rate of the capital stock is *higher* in the 1965-1990 data, but the rate of growth of the labor force is somewhat *lower*. The most striking change is the rate of TFP growth, which falls from 0.0230 to 0.0042, a decrease of almost 2 full percentage points. Thus, the decrease in the growth rate of GDP in Mexico is largely attributed to a decrease in the rate of total factor productivity growth; Mexico has experienced a “productivity slowdown.”