Problem Set #6

Economic Growth
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Consider the model of human capital. Output is produced according to the production function $Y = K^{\alpha}(huL)^{1-\alpha}$, where $0 < \alpha < 1$ and where u is the fraction of time that each person spends working. A constant fraction s of output is invested in new physical capital, so that physical capital accumulation is given by

$$\dot{K}(t) = sK(t)^{\alpha}(h(t)uL(t))^{1-\alpha} - \delta K(t).$$

Human capital accumulation is given by

$$\dot{h}(t) = (1-u)h(t).$$

The labor force $L(t)$ grows at the constant rate $n > 0$.

a) Derive the differential equations for $k = \frac{K}{L}$ and $\widehat{k} = \frac{k}{h}$.

b) Draw the phase diagram for (k, h) and the Solow diagram for $\widehat{k}$. Be sure to label all of the lines and curves in your graphs.

c) We are going to do the following comparative dynamics exercise: $s' > s$.

The exercise takes the usual form. The baseline economy has savings rate s and is on the balanced growth path at $t = 0$. The modified economy starts at $t = 0$ with the same amounts of physical capital and human capital as the baseline economy, but with the savings rate s'.

Draw the modified phase diagram for (k, h) and Solow diagram for $\widehat{k}$, indicating what has changed.

d) Draw the time paths of (the logs of) h, k, and y for both the baseline and the modified economy. Pay particular attention to the slopes of these functions right at $t = 0$.